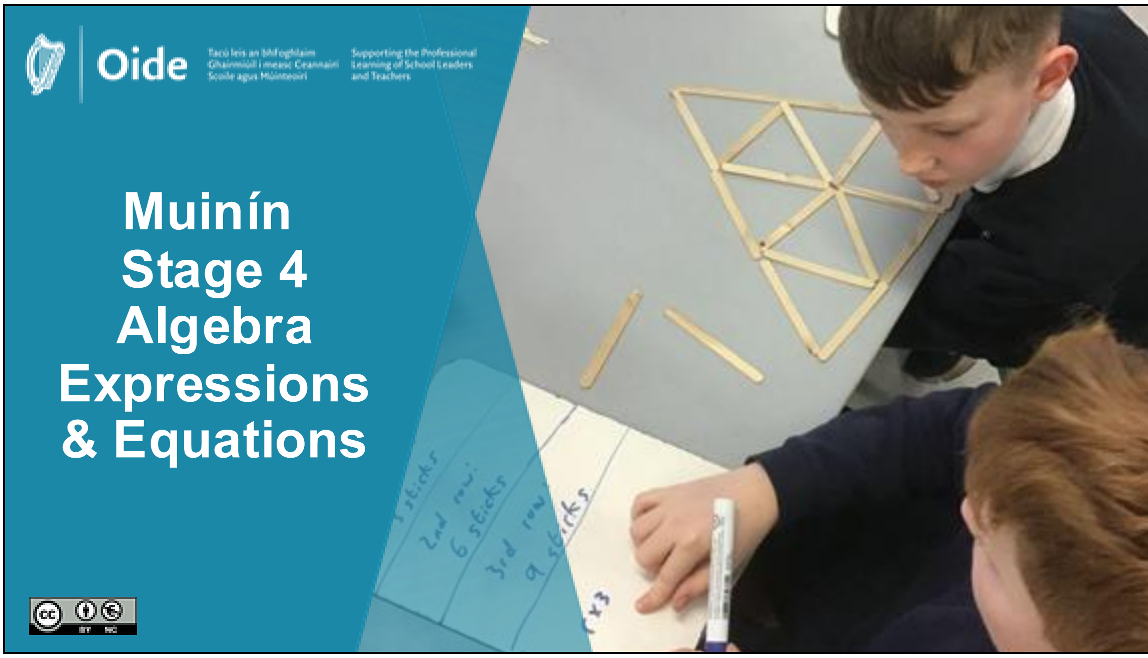
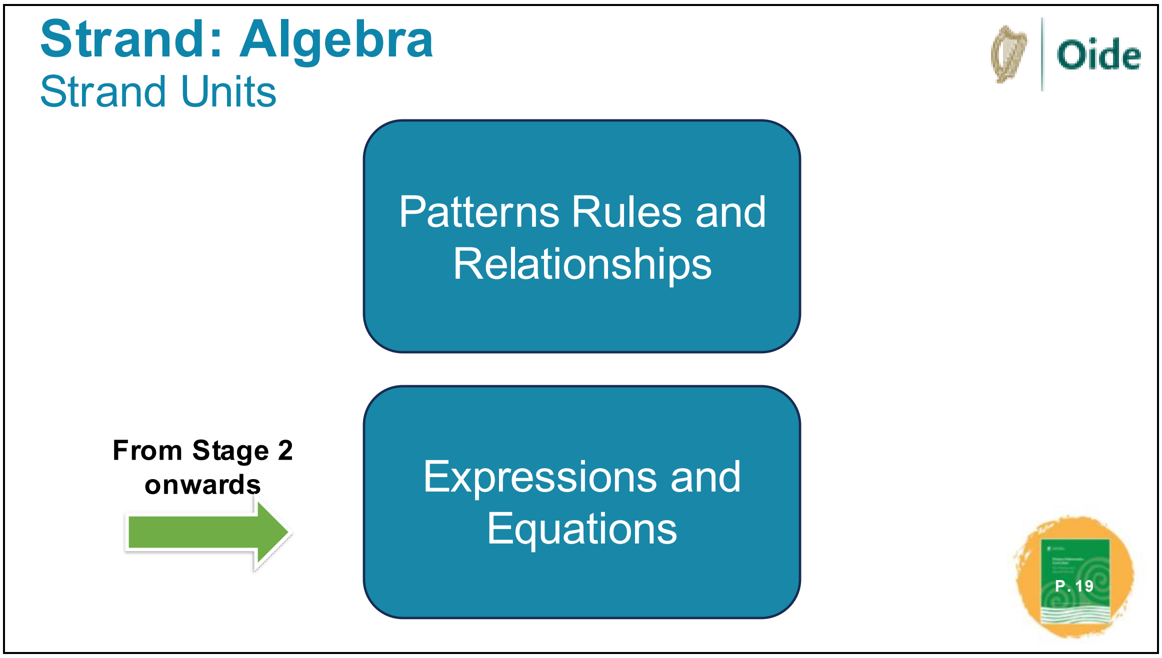


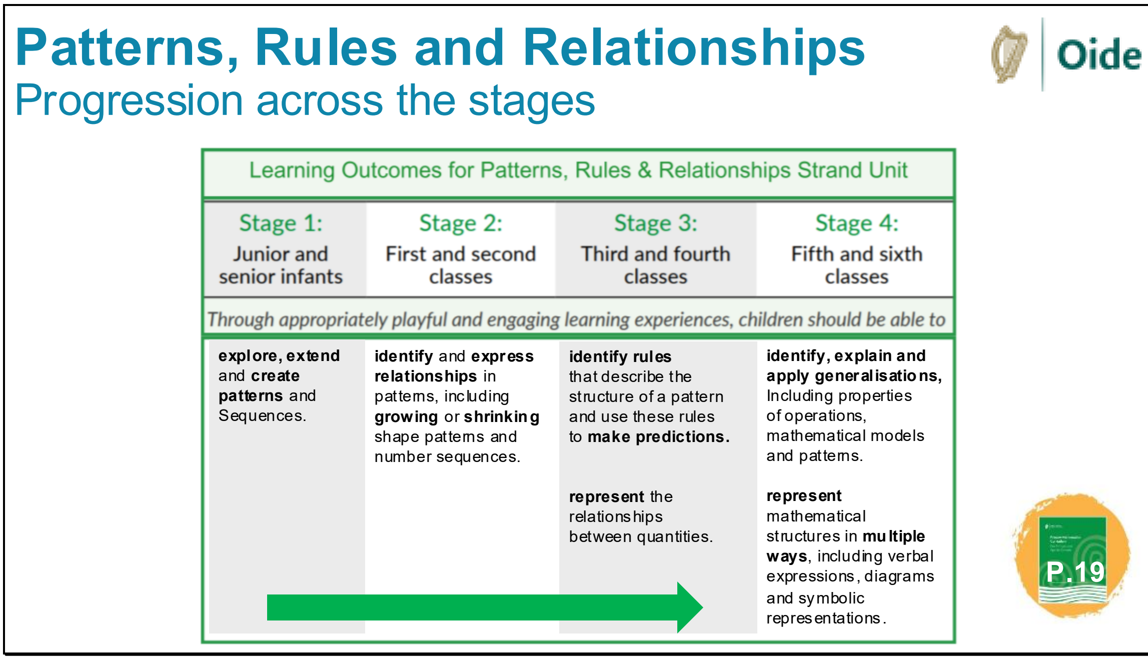
Purpose of slide:
Introductory slide for presentation of Stage 4 Algebra – Expressions and Equations.



Purpose of slide:
To provide an overview of the Algebra strand.

Notes for teachers:

- Go to page 19 of curriculum document.
- There are two strand units within the strand of Algebra.
- **Stage 1 To Stage 4 :** Patterns, Rules and Relationships.
- **From Stage 2 onwards**: Expressions and Equations.
- Why are the strand units presented in this way?
    - Younger students are still developing their abstract thinking skills. The focus is on building a strong foundation in patterns as a great deal of Algebra concerns pattern in one way or another, from making and describing patterns, to making rules and finding general rules to making a rule to predict the results of patterns. Implicit in this is the notion of relationships. As students progress through the curriculum, they will gradually be introduced to algebraic concepts, including expressions and equations, in a way that builds on their existing knowledge and skills.



Patterns, Rules and Relationships

Progression across the stages

Oide

Learning Outcomes for Patterns, Rules & Relationships Strand Unit			
Stage 1: Junior and senior infants	Stage 2: First and second classes	Stage 3: Third and fourth classes	Stage 4: Fifth and sixth classes
Through appropriately playful and engaging learning experiences, children should be able to			
explore, extend and **create patterns** and Sequences.	**identify** and **express relationships** in patterns, including **growing** or **shrinking** shape patterns and number sequences.	**identify rules** that describe the structure of a pattern and use these rules to **make predictions.**	**identify, explain and apply generalisations,** Including properties of operations, mathematical models and patterns.
		represent the relationships between quantities.	**represent** mathematical structures in **multiple ways**, including verbal expressions, diagrams and symbolic representations.

P.19

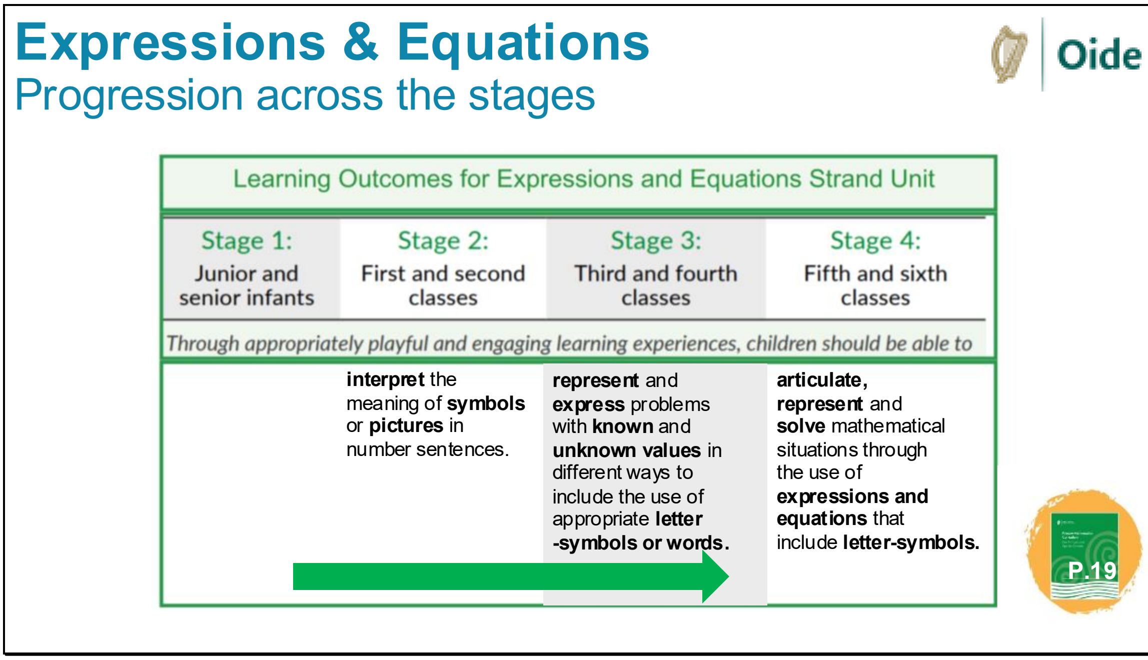
Purpose of slide:
To explore the progression across the stages in the strand unit Patterns, Rules and Relationships.

Notes for teachers:

- Notice the progression along the stages.
- Note how language, knowledge and skills are developed from stages 1 to 4.
- Knowledge of progression is necessary so that we can adapt and extend our teaching based on the knowledge we have of the children in front of us.
- Looking at the learning outcomes we can see how each stage builds upon the last, fostering a rich understanding of symbols and their mathematical significance.
- Algebra permeates all strands of this curriculum and is at the heart of mathematics.
- There are two types of algebra, formal algebra (equations) and algebraic thinking/reasoning.
- We have up until now only encountered formal algebra when

entering post primary school and in general we had no basic understanding/experience of algebraic thinking.

- The emphasis for Algebra in primary school should be on the development of algebraic thinking.
- Algebraic thinking is essential for students to engage meaningfully with formal algebra.



Purpose of slide:
To explore the progression across the stages in the strand unit Expressions & Equations.

Notes for teachers:

- Notice the progression along the stages.
- Note how language, knowledge and skills are developed from stages 1 to 4.
- Knowledge of progression is necessary so that we can adapt and extend our teaching based on the knowledge we have of the children in front of us.
- Looking at the learning outcomes we can see how each stage builds upon the last, fostering a rich understanding of symbols and their mathematical significance.
- Algebra permeates all strands of this curriculum and is at the heart of mathematics.
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- The emphasis for Algebra in primary school should be on the development of algebraic thinking.
- Algebraic thinking is essential for students to engage meaningfully with formal algebra.

Introductory Task

Oide

Always, Sometimes, Never

3 + 4 - 2 = 6	4 + 2e = 6e
3 + 22 = 24 +1	(t+ 5) is less than 20
2n - 5 = 11	2(3 + s) = 6 + 2s

Purpose of slide:
To engage in a maths task that promotes maths talk based on Algebra.

Notes for teachers:

- Always, Sometimes, Never activities are suitable for all age groups. Learners are asked to use their reasoning to decide if mathematical statements are always true, sometimes true or never true. They should then provide examples to prove when statements are always true, sometimes true or never true.
- This will help learners to critically examine mathematical statements and to understand when they apply.
- Key questions could include:
  - *Is this statement always true, sometimes true or never true?*
  - *Why did you think the statement was always, sometimes or never true?*
  - *Did anyone think something different?*
  - *Can you think of a time when this statement is not true?*
  - *Can you prove that it is always, sometimes or never true?*
  - *If the statement is sometimes true, can you give an example of*

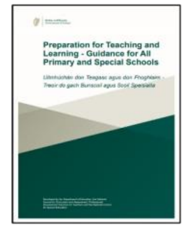
when it is and it isn't true?

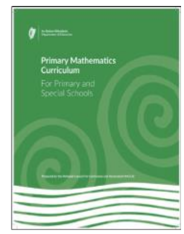
- Use the QR code on the slide to find more examples of Always, Sometimes, Never in the Micromaths section of pmc.oide.ie

Learning Outcome
Recorded Preparation

Oide

Learning Outcome:
Through appropriately playful and engaging learning experiences, children should be able to:
articulate, represent and solve mathematical situations through the use of **expressions** and **equations** that include letter-symbols.





Purpose of slide:
To highlight the learning outcome as the starting point for preparation for teaching and learning.

Notes for teacher:
This is the learning outcome for Stage 4 Algebra (Expressions and Equations)

- Many children enjoy, and experience success in mathematics in primary school, until the introduction and manipulation of abstract symbols in secondary school.
- Algebra is a form of expression; it's a way to communicate.
- If I'm going to communicate something, I need to understand what it is. No more should children be presented with a + b = b + a until they have a robust understanding of equivalence, the underpinning properties (commutative…) and why and how we use symbols.
- This develops over time.
- About 70% of questions in the Junior Cert maths exam requires algebra. This is only 3 years from when they leave us!
- Key Findings of the Early Algebra Movement
    - Understanding Symbols in Maths - Using symbols in

mathematics is like using a language. Students must truly understand what these symbols mean. This means moving beyond just memorizing rules, such as \(a + b = b + a\) or solving for \(x\) in equations like \(3x + 7 = 34\).

- Learning Takes Time - Understanding algebra takes time and practice. Students need many chances to hear, see, and express their ideas about maths.
- What Students Need to Learn - Children should understand the idea of equivalence, which means knowing when two expressions are equal. It’s important for students to know why we use symbols in math and how they help us solve problems.

Maths Concepts
Stage 4

Stage 4 (5th & 6th Class)	
Learning Outcomes	articulate, represent and solve mathematical situations through the use of expressions and equations that include letter-symbols.
Mathematical concepts	A real-life situation can be represented by an expression or a series of expressions.
	An expression may contain more than one unknown or variable. Each unknown or variable must be represented by a dedicated symbol.
	When generating an expression to represent a real-life situation, it can be possible and useful to 'simplify' a long or complex expression.
	'Solving' an equation consists of determining which value(s) for a given symbol make(s) the equation true.

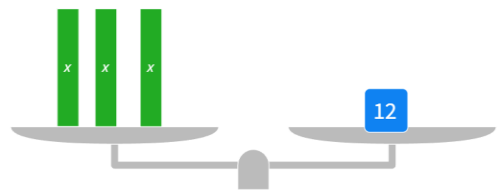
Purpose of slide:
To highlight the Maths Concepts which underpin the learning outcome for Stage 4 Expressions & Equations.

Notes for teachers:
- The Maths Concepts are the key mathematical ideas that underpin each learning outcome.
- The Maths Concepts may be useful in identifying a Focus of New learning when preparing for teaching and learning.
- Take a few moments to explore the Learning Outcomes and the Maths Concepts on the NCCA Maths Toolkit by using the QR code above.

Equivalence

$$8 + 4 = \square + 5$$

- **The Equals sign =**
- **Relational:** the same as or equivalent to.
- **Operational**: Requires some kind of action, e.g., ‘adds up to’, ‘produces’, ‘write something’

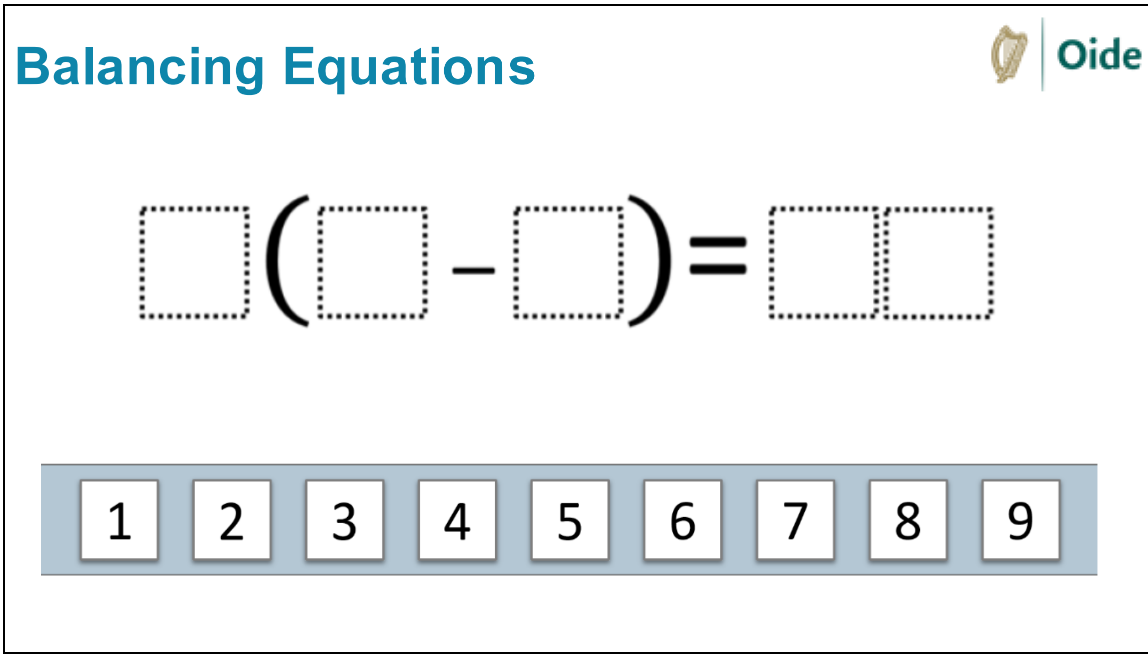


Purpose of slide:
To introduce the concept of equivalence and highlight the importance of language.

Notes for teachers:

- Repeated studies have found that fewer than 10% of children between ages 6-12 correctly complete this equation.
- In one 1999 study, not one of 145 12-year-old participants solved it correctly (Faulkner, Levi & Carpenter, 1999).
- Students are immersed in completing their sums such as 3x8=__; 6+2=__ and it becomes automatic to think “the answer comes next”.
- This misconception is highly problematic when students encounter complex expressions and equations in senior primary and in secondary school.
- The understanding of equivalence is an essential knowledge for algebra. Children must develop a relational understanding of equality.

- Many students interpret the = sign as ‘makes’ or as a signal to ‘find the answer. It is important to emphasise that the = sign means ‘is equal to’ and that it indicates that both sides of the equation represent the same number.
- Use the QR code on the screen or this link https://polypad.amplify.com/p/c7nvBuCCdXaIpw to use the balance on Amplify polypad to help develop this concept.



Purpose of slide:
To engage in a maths task that highlights the relational aspect of the equal sign.

Notes for teachers:
- This is an open-ended task with multiple solutions and multiple solution pathways.
- TASK:
    - Use all the digits above to create an equation. Zero can be added in too.
    - Students can work with a partner/group on the task and use resources of their choice.
    - Teacher promotes maths talk by asking children how they engaged with the task.
    - This task can be adapted/differentiated for different stage levels. For example, __ + ___ = 8 and the children can find the different solutions to the task.
    - The task can be made more challenging by using each digit only once.

Open Middle
Balance Equations

Oide

Balance Equations

Use the operation symbols + − × ÷ and equals sign = to make a true equation.

Operation symbols may be used more than once.

What is the least value for each part of the equation?
What is the greatest value for each part of the equation?

$$2 \square (3 \square 7 \square 9) \square (1 \square 5) \square (8 \square 4)$$

Can you complete the equation with at least one of each symbol?

Adapted from: https://www.openmiddle.com/operation-symbols/

Purpose of slide:
To provide an example of a maths task that highlights the relational aspect of the equal sign.

Notes for teachers:
This is an open-ended task with multiple solutions and multiple solution pathways.

Variables

Purpose of Slide:
To provide an example of a maths task that explores variables and equality.

Notes for teachers:

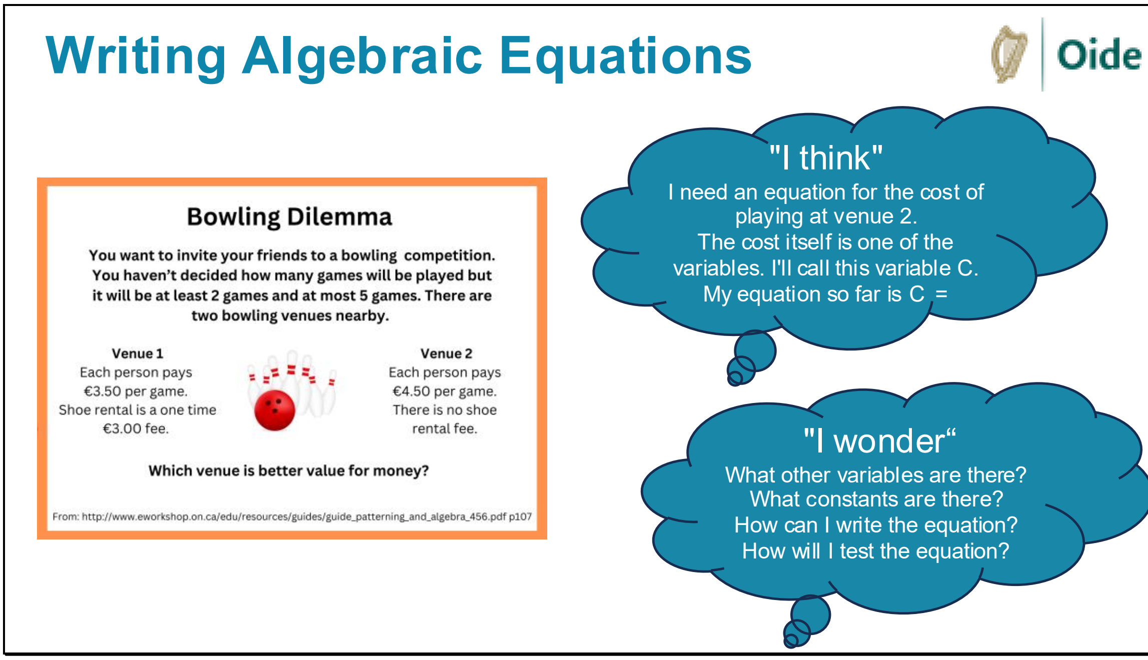
- This is an open-ended task with multiple solutions and multiple solution pathways.
- In this learning activity, students explore the concept of equality and variables.
- In the context of a literature connection (see link to story below), students use symbols and letters to represent the masses of different animals and to investigate relationships and express them algebraically. Students identify relationships between the masses of the animals and write equations that represent balanced tug-of-war teams.
- Tell students to work with their partner to draw a diagram or an equation showing how the masses of the animals in the story are balanced. Ask the class to brainstorm ways they could represent each animal without having to draw each animal. Students may suggest using letters, shapes, or graphics.
- Can pupils produce other equations of equivalence
    - Foxes and squirrels?

- Turtles and squirrels?
- All three in the 1 equation?

- Visualisation will help pupils make a mental picture of what is happening. Concrete materials will aid and assist them to do this.
- As students begin to record their equations, ask students to explain their solutions. Resist the urge to direct students to specific equations. Deeper learning can be achieved by asking questions such as:
    - *How is this side of the equal sign balanced with the other side?*
    - *I'm still not sure I understand your thinking; can you think of another way to show me?*
    - *I see you have drawn pictures to show your equations; is there another way to record your thinking?*
    - *How do you know the equation is true?*
- Continue to observe the students as they work. Call the group back together when you feel the groups have exhausted the solutions, they are able to create.

Resources required:

- "Equal Shmequal" (Book by Virginia L Kroll) is a great resource for exploring equivalence.
  A read aloud is available here https://www.youtube.com/watch?v=i_-kd0Lv-5E



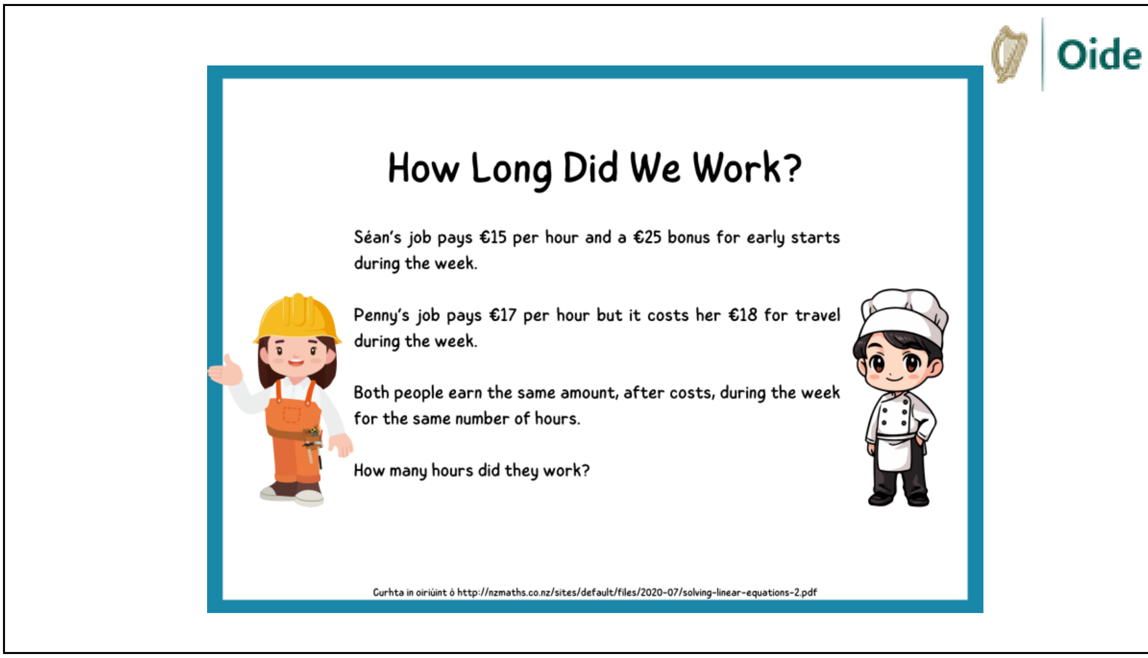
Purpose of slide:
To provide an example of a maths task that explores writing algebraic equations that include letter symbols.

Notes for teachers:

- In this learning activity, students use graphs, tables of values, and equations to compare costs at two different bowling sites, and to determine which site offers the better deal.
- Using a think-aloud method, pose the following statements to the class before the students begin working on the problem:
  - **"I think…."**
    - I need an equation for the cost of playing bowling at both venues.
    - The cost itself is one of the variables. I'll call this variable C.
  - **"I wonder…."**
    - What other variables are there? What letters could I use to represent them?
    - What constants are there?
    - How can I write the equation?

- How will I test the equation?

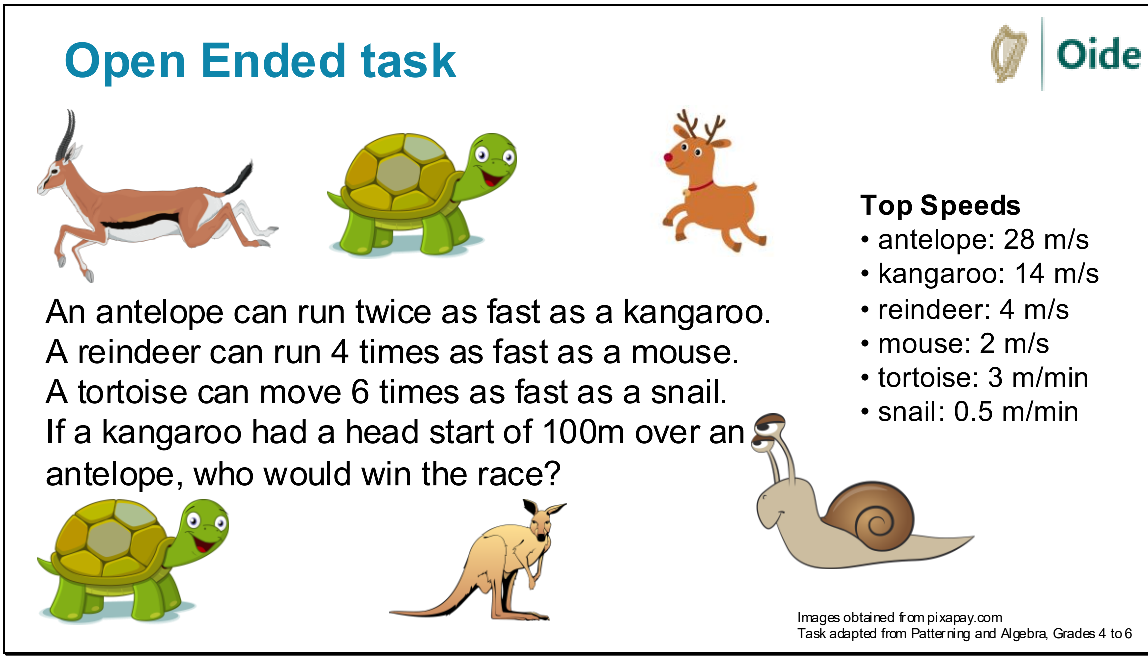
- Some suggestions for equations:
  - Venue 1: C = €3.50(g) + €3.00   C = Cost, g = number of games.
  - Venue 2: C = €4.50(g)
- How will we check which venue is the best value for 2,3,4 or 5 games played?



Purpose of slide:
Introduce a task available on the PMC hub (www.pmc.oide.ie).

Notes for teachers:

- Similarly to the previous task, encourage students to use graphs, tables of values, and equations to discover how many hours Seán and Penny worked noting that the number of working hours are not provided.
- Some suggestions for equations:
  - <u>Sean:</u> €15 (h) + €25
  - <u>Penny:</u> €17 (h) - €18
  - So, €15 (h) + €25 = €17 (h) - €18



Purpose of slide
To explore an open ended task based on algebra.

Notes for teachers:

- Think about this task:
    - *What makes this task an open ended task?*
    - *Is there more than one solution? (multiple pathways and multiple solutions)*
    - *Is there a set of rules or procedures to follow?*
    - *What is the mathematical point of the task?*
- Students can draw, use concrete materials or digital manipulatives e.g. Amplify Polypad to complete the task.
- Students predict the running speeds of each animal.
- Reveal the relationships between the running speeds of the animals and ask students to compare it to their prediction:
    - *A pronghorn antelope can run twice as fast as a kangaroo.*
    - *A reindeer can run 4 times as fast as a house mouse.*
    - *A giant tortoise can move 6 times as fast as a garden snail.”*
- Ask a question: “*Suppose that an antelope and kangaroo had a race, and*

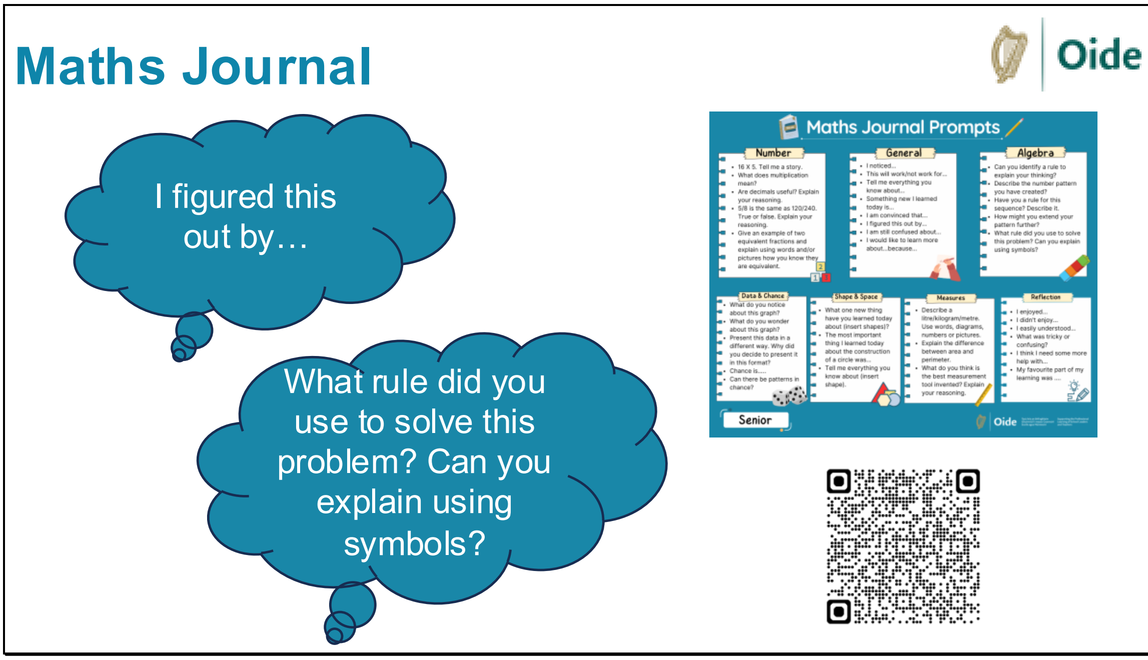
the kangaroo had a head start of 100 m. Who would win the race?"

- Ask students to work in pairs on the problem, then discuss solution ideas as a whole class.
- Draw attention to the various methods that students use to analyse the problem, such as drawing a diagram, making a T-chart, or acting out the situation (see below).
- Students will notice that the length of the race has not been specified.
- Ask them to consider different race lengths to see if length makes a difference i.e. distance

Time taken = Distance/Speed

	100m	200m	300m	400m	500m
Antelope	100/28	200/28	300/28	400/28	500/28
Kangaroo (100m headstart)	0	100/14	200/14	300/14	400/14

- Can pupils see a pattern?
- What will happen over a longer period of time?
- Could we draw up a table of other races between other animals?



Purpose of slide:
To provide reflective prompts to use in class.

Notes for teachers:

- Journals are useful for both teachers and learners to assess attitudes, knowledge and skills.
- Children can keep track of their thinking and understanding in the journal.
- Journals can contain general observations about Maths or can be more specific and focus on a particular concept.
- On the slide are two journal prompts which can be used in class. The first one focuses on the child's disposition and can be used across all strand units.
- Journal prompts:
    - I OBSERVED, I DISCOVERED, I CHECKED, I PROVED…
    - Today I enjoyed… general prompt to get the children thinking about maths and the areas that they are curious about.
    - Specific strand-based prompts….How many ways can you make €1? Use words or drawings to explain your thinking.

- Use the QR Code on the slide to find the above journal prompts on the PMC Hub.