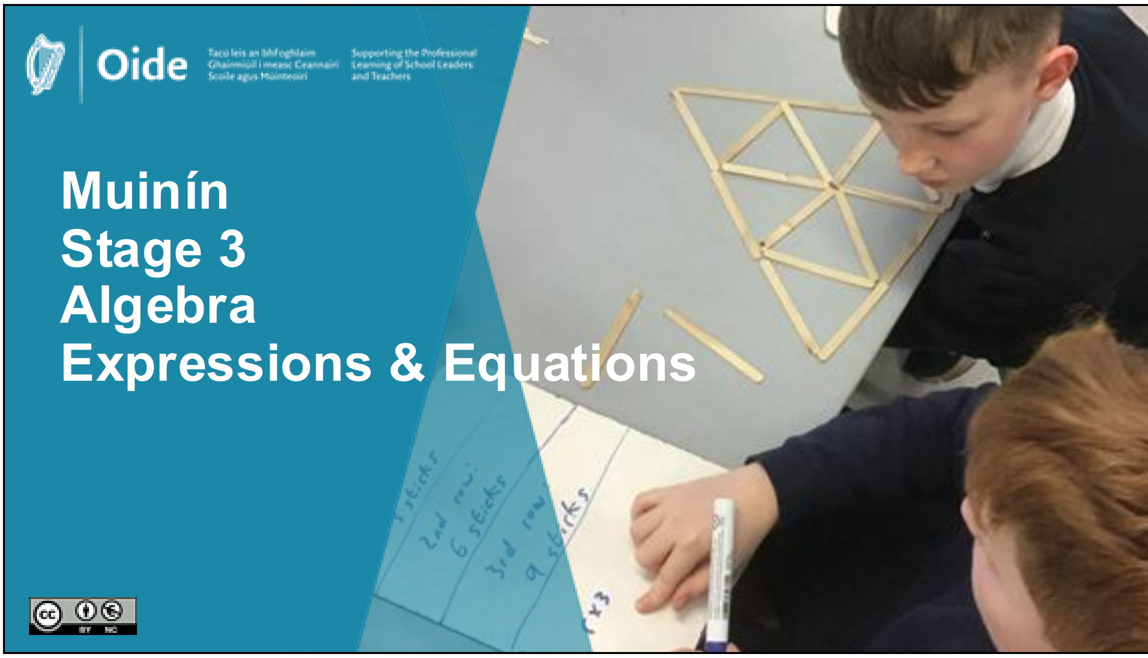
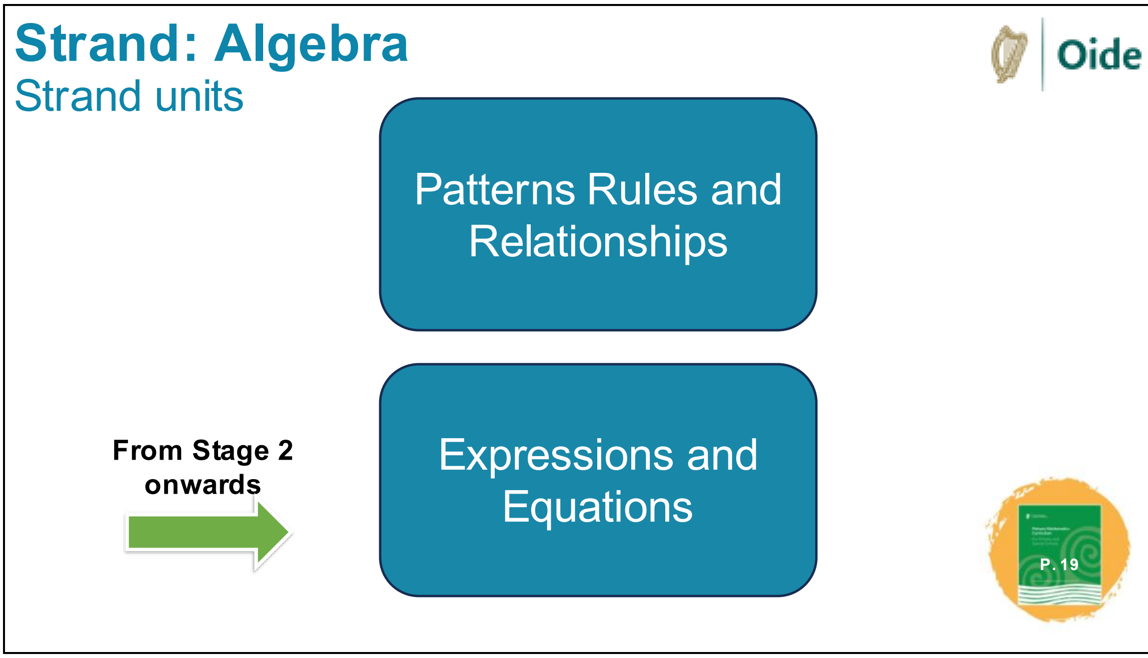


Purpose of slide:
Introductory slide for presentation of Stage 3 Algebra – Expressions and Equations.



Purpose of slide:
To provide an overview of the Algebra Strand.

Notes for teachers:

- Algebra is a useful tool for generalising number patterns and representing patterns.
- Symbols such as the equals sign must be well understood conceptually for children to be successful in mathematics.
- Methods to carry out the operations can be generalised e.g. a+b=b+a
- Conceptual understanding of generalisations can be strengthened through the use of multiple representations as each representation provides the same view of a different relationship.
- There is a strong link between Algebra and all other stands of the PMC: Number, Data, Measure and formulas for measuring, Shape and Space (patterning).

Patterns, Rules and Relationships

Progression across the stages

Oide

Learning Outcomes for Patterns, Rules & Relationships Strand Unit			
Stage 1: Junior and senior infants	Stage 2: First and second classes	Stage 3: Third and fourth classes	Stage 4: Fifth and sixth classes
Through appropriately playful and engaging learning experiences, children should be able to			
explore, extend and **create patterns** and Sequences.	**identify** and **express relationships** in patterns, including **growing** or **shrinking** shape patterns and number sequences.	**identify rules** that describe the structure of a pattern and use these rules to **make predictions.** **represent** the relationships between quantities.	**identify, explain and apply generalisations,** Including properties of operations, mathematical models and patterns. **represent** mathematical structures in **multiple ways**, including verbal expressions, diagrams and symbolic representations.

Purpose of slide:
To explore the progression across the stages in the strand unit Patterns, Rules and Relationships.

Notes for teachers:

- Notice the progression along the stages.
- Note how language, knowledge and skills are developed from stages 1 to 4.
- Knowledge of progression is necessary so that we can adapt and extend our teaching based on the knowledge we have of the children in front of us.
- Looking at the learning outcomes we can see how each stage builds upon the last, fostering a rich understanding of symbols and their mathematical significance.
- Algebra permeates all strands of this curriculum and is at the heart of mathematics.
- There are two types of algebra, formal algebra (equations) and algebraic thinking/reasoning.
- We have up until now only encountered formal algebra when

entering post primary school and in general we had no basic understanding/experience of algebraic thinking.

- The emphasis for Algebra in primary school should be on the development of algebraic thinking.
- Algebraic thinking is essential for students to engage meaningfully with formal algebra.

Expressions & Equations

Progression across the stages

Oide

Learning Outcomes for Expressions and Equations Strand Unit			
Stage 1: Junior and senior infants	Stage 2: First and second classes	Stage 3: Third and fourth classes	Stage 4: Fifth and sixth classes
Through appropriately playful and engaging learning experiences, children should be able to			
	interpret the meaning of **symbols** or **pictures** in number sentences.	**represent** and **express** problems with **known** and **unknown values** in different ways to include the use of appropriate **letter -symbols or words.**	**articulate, represent** and **solve** mathematical situations through the use of **expressions and equations** that include **letter-symbols.**

P. 19

Purpose of slide:
To explore the progression across the stages in the strand unit Expressions & Equations.

Notes for teachers:

- Notice the progression along the stages.
- Note how language, knowledge and skills are developed from stages 1 to 4.
- Knowledge of progression is necessary so that we can adapt and extend our teaching based on the knowledge we have of the children in front of us.
- Looking at the learning outcomes we can see how each stage builds upon the last, fostering a rich understanding of symbols and their mathematical significance.
- Algebra permeates all strands of this curriculum and is at the heart of mathematics.
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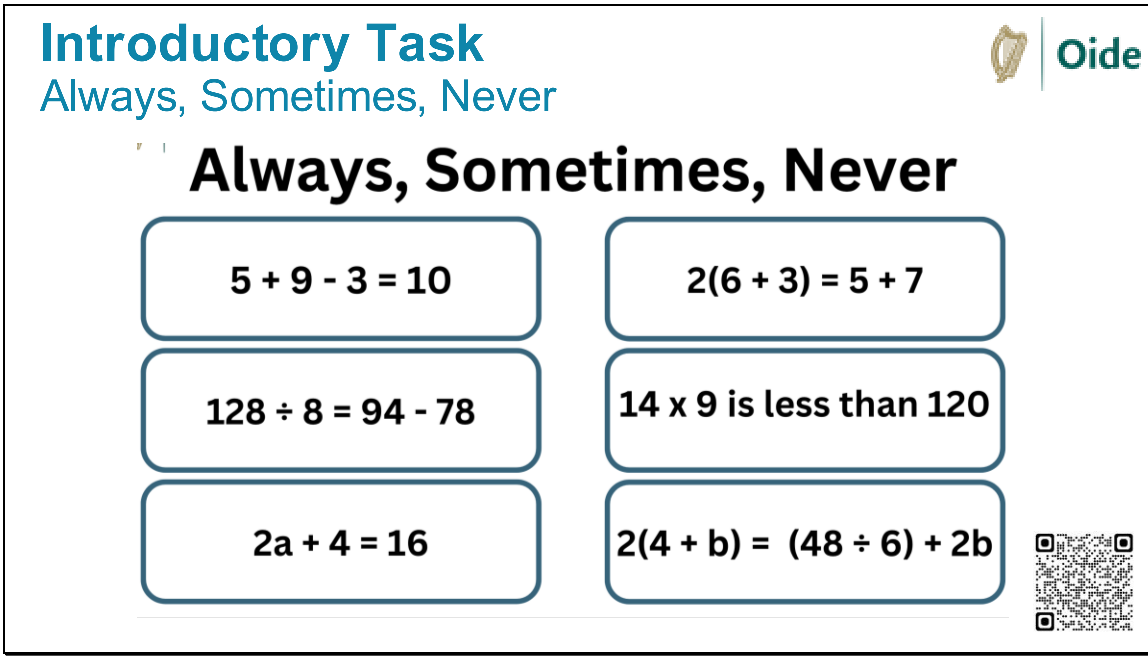
- The emphasis for Algebra in primary school should be on the development of algebraic thinking.
- Algebraic thinking is essential for students to engage meaningfully with formal algebra.



Purpose of slide:
To engage in a maths task that promotes maths talk based on Algebra.

Notes for teachers:

- Always, Sometimes, Never activities are suitable for all age groups. Learners are asked to use their reasoning to decide if mathematical statements are always true, sometimes true or never true. They should then provide examples to prove when statements are always true, sometimes true or never true.
- This will help learners to critically examine mathematical statements and to understand when they apply.
- Key questions could include:

- *Is this statement always true, sometimes true or never true?*
- *Why did you think the statement was always, sometimes or never true?*
- *Did anyone think something different?*
- *Can you think of a time when this statement is not true?*
- *Can you prove that it is always, sometimes or never true?*
- *If the statement is sometimes true, can you give an example of when it is and it isn't true?*

- Use the QR code on the slide to find more examples of Always, Sometimes, Never in the Micromaths section of pmc.oide.ie

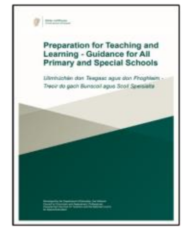
Learning Outcome

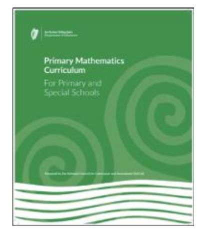
Recorded preparation

Learning Outcome

Through appropriately playful and engaging learning experiences, children should be able to:

represent and express problems with known and unknown values in different ways to include the use of appropriate letter-symbols or words.





Purpose of slide:
To highlight the learning outcome as the starting point for preparation for teaching and learning.

Notes for teacher:

- This is the learning outcome for Stage 3 Algebra (Expressions and Equations)
- Many children enjoy, and experience success in mathematics in primary school, until the introduction and manipulation of abstract symbols in secondary school.
- Algebra is a form of expression; it’s a way to communicate.
- If I’m going to communicate something, I need to understand what it is. No more should children be presented with a + b = b + a until they have a robust understanding of equivalence, the underpinning properties (commutative…) and why and how we use symbols.
- This develops over time.
- About 70% of questions in the Junior Cert maths exam requires algebra. This is only 3 years from when they leave us!
- Key Findings of the Early Algebra Movement

- Understanding Symbols in Maths - Using symbols in mathematics is like using a language. Students must truly understand what these symbols mean. This means moving beyond just memorizing rules, such as \(a + b = b + a\) or solving for \(x\) in equations like \(3x + 7 = 34\).
- Learning Takes Time - Understanding algebra takes time and practice. Students need many chances to hear, see, and express their ideas about maths.
- What Students Need to Learn - Children should understand the idea of equivalence, which means knowing when two expressions are equal. It's important for students to know why we use symbols in math and how they help us solve problems.

Maths Concepts

Recorded preparation

Oide

When expressing real-life situations, symbols can be used to represent an unknown, a quantity that varies (*variable*), or every number (*the general case*).

Representing the structure of a function using words, symbols, graphs, tables or diagrams is useful to identify outputs for given inputs and vice versa.

Real-life situations and functions can be represented in a variety of forms, including numbers, words, symbols and tables.

A function is a special relationship where each input has exactly one output. There are always three main parts; the input, the functional relationship and the output.

Purpose of slide:
To highlight the Maths Concepts which underpin the learning outcome for Stage 3 Expressions & Equations.

Notes for teachers:

- The Maths Concepts are the key mathematical ideas that underpin each learning outcome.
- The Maths Concepts may be useful in identifying a Focus of New learning when preparing for teaching and learning.
- Take a few moments to explore the Learning Outcomes and the Maths Concepts on the NCCA Maths Toolkit by using the QR code above.

Vocabulary

Variable: a number that is liable to change and not consistent.

Function: defines the relationship between one variable and another.

The input and the output: the input is the number you feed into an expression, and the output is what you get after the calculations are finished.

Purpose of slide:
To introduce the mathematical language of variable, function, input and output.

Notes for teachers:

- Some definitions that you could share with the children.
- You could give examples such as:
    - A variable is something that is not always the same, elicit from the children do they know anything that they would consider a variable? For example a person's eye colour, hair colour, they are variable. Even the temperature in the room can be variable, it is not always the same.
    - Functions are used to perform single or sequential tasks, they can be short and simple or long and complicated. Functions can be reused and adapted. Essentially, functions can do anything you tell them to. Explaining functions to kids is easy, just think of things they do every day, like brushing their teeth

- An input is whatever you put into a system. An output is whatever comes out of the system. For example, a computer has inputs like electricity, the movements and clicks of your mouse, and the keys you type on a keyboard.
- Some examples of output include: Sound coming from a mobile phone speaker. Text appearing on a laptop screen or computer monitor. Pictures printed from a printer.

Equivalence

$$8 + 4 = \square + 5$$

Operational view of Equality

8 + 4 = 12 + 5
8 + 4 = 17 + 5

Relational view of Equality

8 + 4 = 7 + 5

Purpose of slide:
To introduce a learning experience.

Notes for teachers:

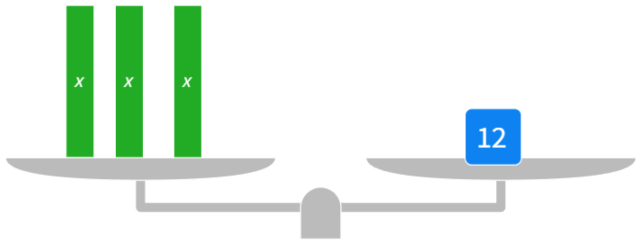
- Repeated studies have found that fewer than 10% of children between ages 6-12 correctly complete this equation.
- In one 1999 study, not one of 145 12-year-old participants solved it correctly (Faulkner, Levi & Carpenter, 1999).
- Students are immersed in 3x8=__; 6+2=__ and it becomes automatic to think “the answer comes next”.
- This misconception is highly problematic when students encounter complex expressions and equations in senior primary and in secondary school.
- To investigate your pupils’ understandings of equality, present them with the above problem and record their responses.
- The above (incorrect) responses indicate an operational view of equality,

eg:

  - 8 + 4 = 12 + 5
  - 8 + 4 = 17 + 5

- When children give responses of 12 or 17, they believe that the equals sign means the answer is. Consequently, it seems logical to such children that when you see an equals sign you should perform the calculation that precedes the equals sign and that the number to the right of the equals sign is the answer to that calculation.
- This ‘operator’ view is the result of an overemphasis on problems of the form a + b = q
- This correct response indicates a relational view of equality: 8 + 4 = 7 + 5
- Children with a relational view of equality believe that the equals sign means ‘is the same value as’. Such children understand that the amounts either side of the equals sign are relationally the same. They then search for a value that when placed in the frame will result in both sides of the equals sign balancing each other.

Equivalence

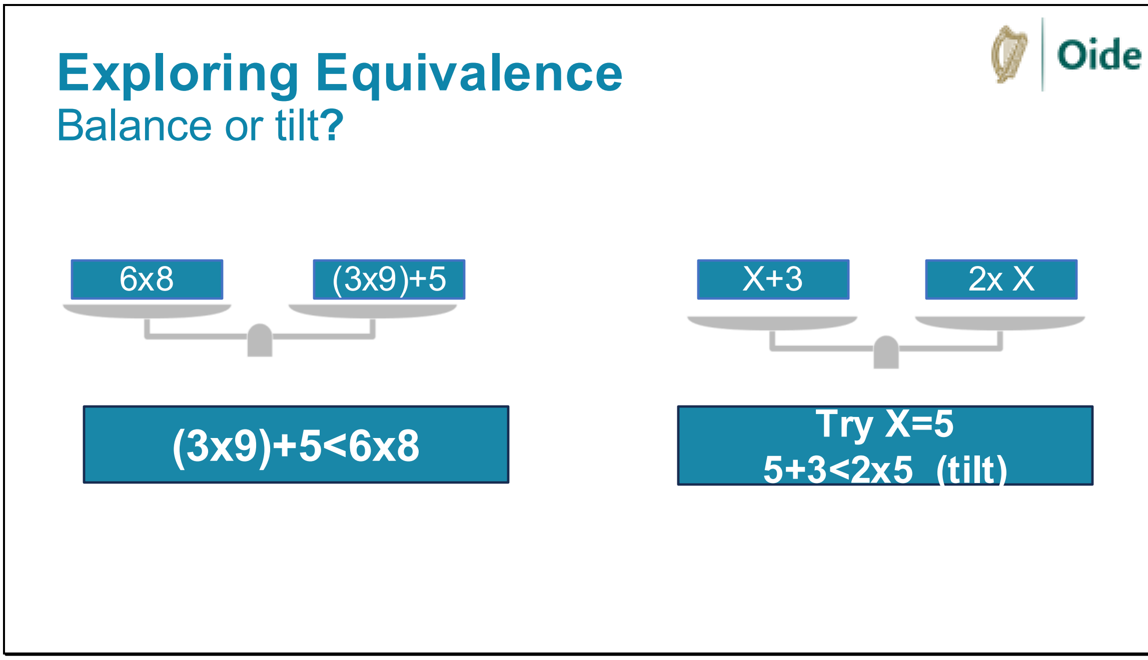


Purpose of slide:
To highlight the importance of language .

Notes for teachers:

- It is important to emphasise that the = sign means ‘is equal to’ and that it indicates that both sides of the equation represent the same number.
- The Balance scale is a great way to concretely show equivalence.

Resources:
https://polypad.amplify.com/p/c7nvBuCCdXalpw#balance



Purpose of slide:
To highlight the importance of language and engage with an activity to explore equivalence using expressions and equations.

Notes for teachers:

- Use a two-pan balance on the whiteboard. Write an expression on one side and another expression on the other side of the balance (image 1). Children must work out if the balance will be equal or tilt. Here they can use a range of strategies to solve the problem. If the balance is equal, they write the equation with the = sign. If the balance is titled, children use the < and > signs.
- Extension- Write one expression on either pan balance. Children must write an expression on the other to balance the scales. A range of correct solutions will be given here and can be explored.
- When children are very comfortable with using the balance and writing equations, we can now bring in the unknown value of X or just a box.

- Here, children find the value for x by trialling different numbers and again, writing expressions for each.

True or False

8 x 6 is equal to 240 ÷ 5	12 x 9 is equal to 649 - 541	38 x 21 = 42 x 15
29 x 6 = 2, 088 ÷ 12	3 x 6 x 5 = 15 x 6	7 x 9 is equal to 504 ÷ 8
1,876 ÷ 67 is equal to 868 ÷ 31	12 x 12 is equal to (11 x 12) + 12	5 x 8 x 3 = 10 x 2 x 6
19 x 56 = 28 x 38	5^2 x 5 x 3 = 25 x 15	24 x 13 is equal to (19 x 11) + (8 x 12) + 7

Adapted from *Elementary and Middle School Mathematics* John A. Van De Walle

Purpose of slide:
To introduce True or False Sentences as a way of working on expressions and equations.

Notes for teachers:

- Children may not have experience with true/false number sentences, but it is relatively easy to introduce them.
- True or false statements are a good starting point to help students with the equal sign by exploring equations as either true or false.
- Begin with simple examples to explain what is meant by a true equation and what is meant by a false equation.
- Engage children in a general discussion about true/false number sentences or what it means for a number sentence to be true or false
- Provide an example and ask whether the number sentence is true or false. It’s a good idea to start with simple sentences that involve relatively simple

calculations with a single digit to the right of the equal sign e.g. 5+6= 11 etc. Introduce less familiar ones such as 7= 2+5. Students often believe there has to be an operation on the left side and an answer on the right.

- When children are comfortable with these, you can then move onto equations for example 3x7=7x3 is this true or false? How do we know?
- Children check answers using concrete materials. Listen to their reasoning and perhaps plan more statements accordingly. You can also use the balance here to prove statements (explored in the slides previous).
- After children have ample time working with true or false statements, they can write some of their own. Encourage children to write 4 true or false sentences, at least one being true and at least one being false. Encourage them to write a "tricky" one.

Open Number Sentences

Oide

5+11= ____	4+____=16	11+16=___ -1
6-n=7-4	N+5=5+8	15+27= n +28
3x7=7x___	___x4=48	_____= 23-5

Adapted from *Elementary and Middle School Mathematics John A. Van De Walle*

Purpose of slide:
To introduce Open Number sentences as a continuation of true or false sentences.

Notes for teachers:

- Once students have experienced true or false statements, introduce open number sentences.
- These are expressions with a box to be filled in or a letter to be replaced e.g. x, n etc.
- To develop an understanding of open number sentences, encourage students to look at the number sentence as a whole and discuss using words or pictures what the equation represents.
- Again, begin with a simple sentence such as 5+8= __ . Use sentences that are similar to the true or false sentences from previous slide.
- Go through examples on the board with the pupils. Ask them to complete

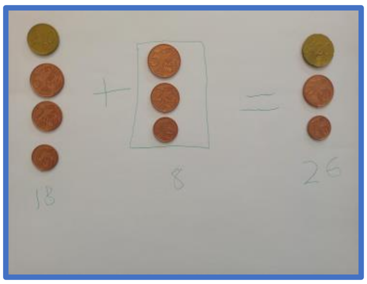
one or two and to explain their thinking/ show their thinking.

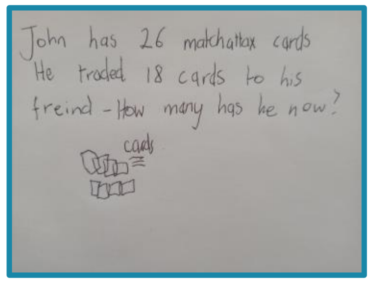
- These sentences can include all 4 operations.
- Allow children to write some open number sentences of their own for their partners to solve.
- Again, children can work concretely, pictorially and abstractly on all tasks (using concrete materials, drawing pictures or working with signs and symbols).

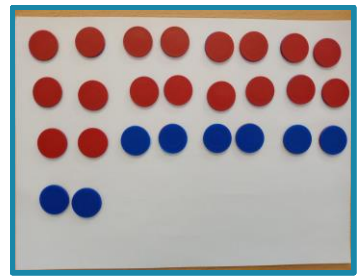
Open Number Sentences

Draw or write an equation

$$18 + n = 26$$







Purpose of slide:
To introduce equations.

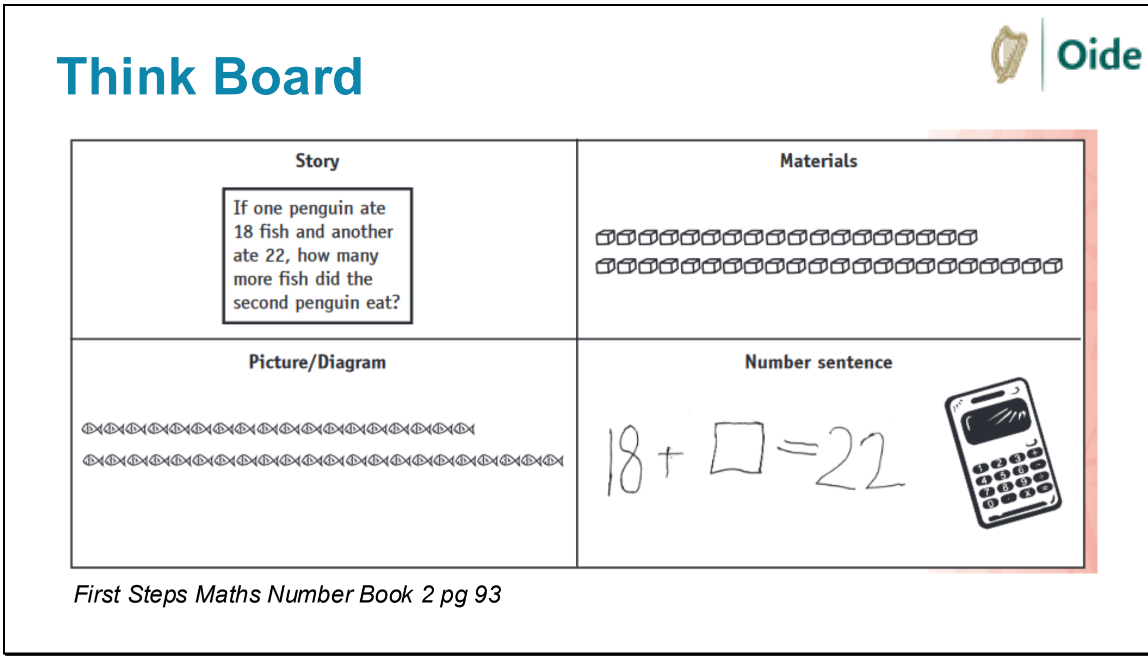
Notes for teachers:

- Here children are encouraged to write or draw their own equation.
- Firstly give children an example of an equation, such as 18 + number = 26.
- You can encourage children to use concrete materials to demonstrate their understanding.
- Here you can see some of the children's work samples. One child has used coins to show their understanding, another child has written a word problem and another has used double sided counters.
- Possible extension opportunities:
    - Ask children to represent their work differently, for example with the coins, what other combinations can be used to make the number 18? Etc.

    - Ask children to form their own equation and ask their peer to solve the equation.

- The use of Mathematical modeling is evident in this task. Modeling involves children using Mathematics to describe a problem-context and determine meaningful solutions to the problem.

- Children form models through a process of testing, revising and expressing their interpretation. Children naturally generate their own informal mathematical models in a way that is context-specific and makes sense to them. As children's knowledge, understanding and experience grows, they may develop more formal, sophisticated and efficient models which they can use to share, connect and communicate their ideas with others. They may also transfer these models to a range of different contexts in a way that is meaningful to themselves and others.



Purpose of slide:
To introduce think boards.

Notes for teachers:

- Students need to develop a deep understanding of the meaning and use of the four basic operations, of their links to each other and to real world applications. In order to build up conceptual links between the various types of situations and the addition, subtraction, multiplication and division operations, a rich and flexible variety of representations is needed over an extensive period of time. It should not be rushed.
- These various forms of represenation include experience-based scripts of real world events or drmatic play, manipulatives, pictures and diagrams, spoken language, written symbols in number sentences. Thus, students should describe, represent and explore both additive (addition and subtraction) and multiplicative (multiplication and division) situations

dramatically, physically, verbally and sumbolically.

- Many teachers have found a a 'think board' helps students to link various ways of representing the operations. There are many different versions of, and ways to use a think board. With the simple versions, one problem is dealt with at a time.
- Think boards can be used to solve Algebra Equations and open number sentences.
- The focus of the think board is on finding the connections between the different ways of representing the problem. This helps students to focus on the meaning of the operation rather than on just calculating to find an answer.

Expressions and Equations

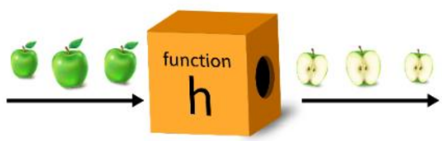
Functions

In mathematics, a function is a mathematical object that produces an output, when given an input. A function can also be called a rule.

Input

Output

Function/Rule

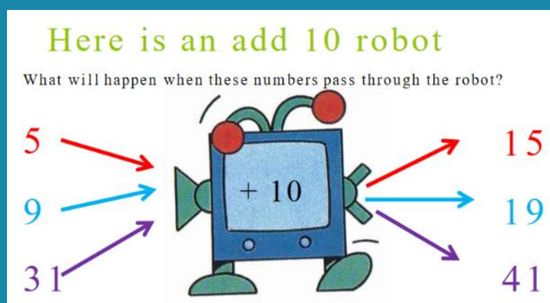


Purpose of slide:
To highlight the definition of functions in mathematics

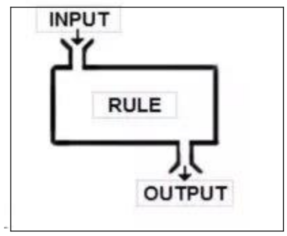
Notes for teachers:

- A function is a mathematical object that produces an output.
- Explaining the concept of a function to a child can be done in a simple and relatable way. You can use the analogy of a vending machine, where you put in a certain type of coin or bill (the input), and then the machine gives you a specific snack or drink (the output).
- Just as different inputs result in different outputs from the vending machine, a function takes an input and gives a specific output. You can also use examples from everyday life, such as how adding numbers in a calculator or following a recipe to bake cookies are both examples of functions.

The "Add" Robot
Activity



Maths Concept:
Real-life situations and functions can be represented in a variety of forms, including numbers, words, symbols and tables.



Purpose of slide:
To show the participants an example of a maths activity using functions.

Notes for teachers:

- Learning outcomes are broad in nature. When working with learning outcomes it is useful to break down the learning outcome into areas of focus. In the case of this activity the learning outcome is represent and express problems with known and unknown values in different ways to include the use of appropriate letter-symbols or words.
- The maths concept is recorded at the top of this slide: Real-life situations and functions can be represented in a variety of forms, including numbers, words, symbols and tables.
- In this activity you can question the children about this robot. What happens when the numbers go through the robot? Will they get larger/smaller? How do you know? What is the input here; it is the numbers 5,9, 31 and function is add 10 and the output is 15,19 and 41.
- You could change this activity and make the function a multiplication

function or a division function.

- It is important to draw the children’s attention to the mathematical language, input, function/rule and output.

The Faulty Oven
Task

Oide


NCE-MSTL
Teaching Algebra in the Primary Classroom: Functions
The 'Function' Machine

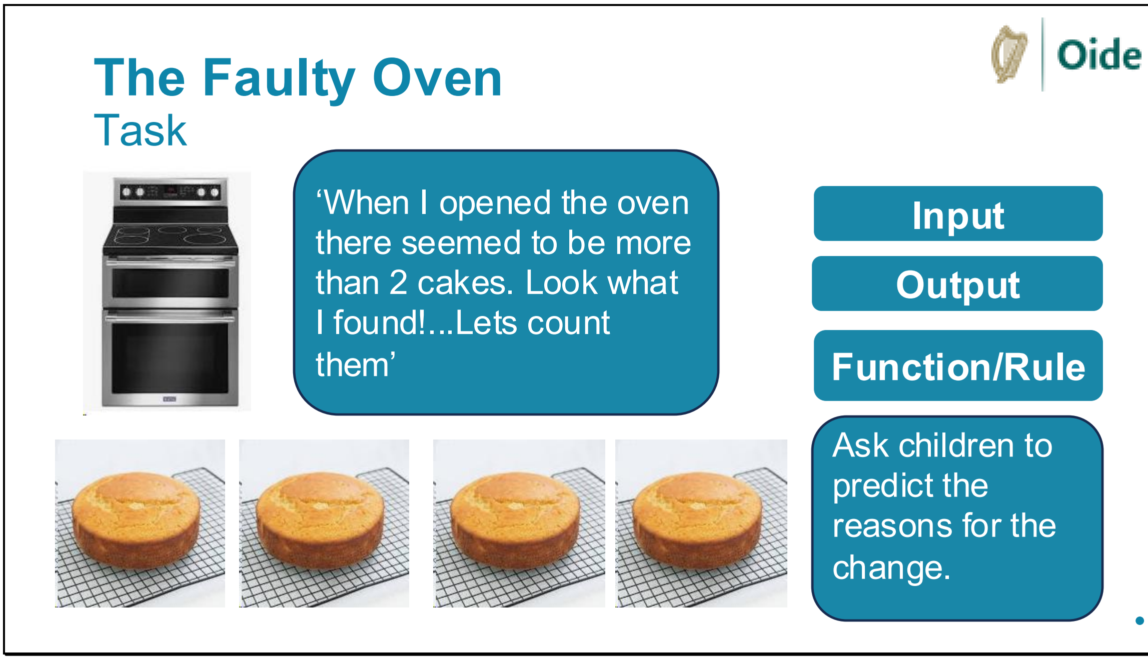
‘Yesterday something very strange happened to me! Well… I love baking and yesterday I made 2 cakes and put them into the oven. When I took the cakes out something strange had happened to them’ Invite the children to respond, pose questions; What do you think happened?

Language: Input, function, output

Purpose of slide:
To introduce the lesson of ‘The Faulty oven’.

Notes for teachers:

- The following scenario is presented to the children: ‘Yesterday something very strange happened to me! Well… I love baking and yesterday I made 2 cakes (picture/photo displayed on the white board using projector) and put them into the oven. When I took the cakes out something strange had happened to them’.
- Invite the children to guess what might have happened.
  <u>Expected/Actual Pupil Responses</u>
  - You put too much baking powder in.
  - They were burnt.
- (Note: This task is adapted from an article by: Hourigan, M., Leavy, A.M. & McMahon (2012) ‘Teaching Algebra in the Primary Classroom. Functions. The Function Machine’).



Purpose of slide:
To develop the lesson of ‘The Faulty oven’.

Notes for teachers:

- <u>Present the actual finding</u>

  ‘When I opened the oven there seemed to be more than 2 cakes. Look what I found! (Show the picture/photo of 4 cakes to the class)...Lets count them’.

  Ask children to predict the reasons for the change.

  <u>Expected/Actual Pupil Response</u>

  - It added 2.
  - It doubled them.
  - It’s like a cloning machine.
  - That must be a special oven.

- The pupils are subsequently given further opportunities to experience the function/rule that the oven is using through other examples:

  ‘Later yesterday evening I made 5 lasagnes (displayed on the board) because I was having friends over for dinner, but when they were cooked I opened the oven and guess what I found?'

Expected/Actual Pupil Responses

- More lasagnes.
- 7 lasagnes.
- 10 lasagnes.

Prompt/probe where necessary e.g. 'Any other guesses?'; 'Did more or less come out do you think?'

The teacher presents the findings and encourages pupils to identify the rule/pattern which determines the number of items e.g. 'I found 10 lasagnes! I wonder what is wrong with my
oven. What is it doing to the food each time?'

Prompt/probe if necessary e.g. 'Did you notice anything special happening to the food that I put in each time?'; 'Do you notice a pattern?'

Expected/Actual Pupil Responses

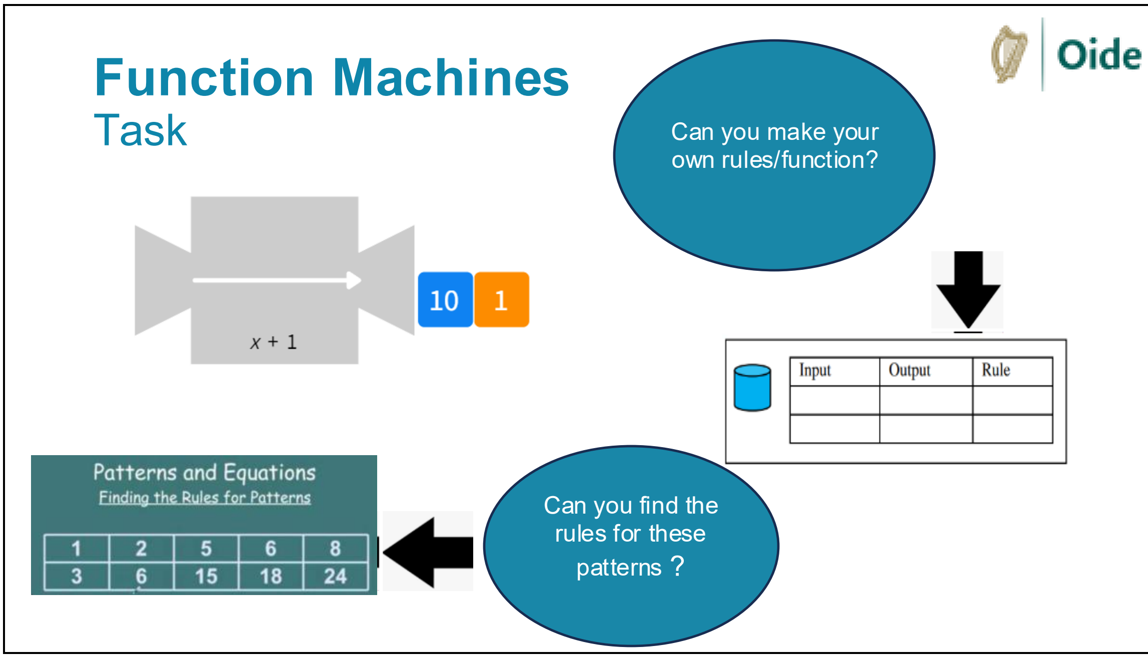
- I wish I had an oven like that!
- It is making the number in the oven bigger each time.
- It is doubling.

Pupils' understanding of the nature of the function/rule can subsequently be tested through teacher questions such as:

'If I were to bake 3 apple tarts in the oven I wonder how many would be there when I would go to take them out?'

- Pupils are introduced to the concept of a function/rule machine:

  'I soon realised that my oven isn't an oven at all…It's a magic machine called the 'Maths Rule Generator'.



Purpose of slide:
To the function machine on Amplify Polypad.

Notes for teachers:
Tell the children that on the slide is a picture of a function machine. You could have a physical machine for the children in the class.
Pupils are given the opportunity to work in groups to generate and identify rules/functions. This can also be done on Amplify Polypad, and the rule can be hidden.

- Following teacher demonstration, pupils will take turns acting as the 'rule generator'. This involves pupils in the group providing inputs (between 0 and 10) and the 'rule generator' working out and sharing the output (using a secret rule e.g. +5).
- Rules for turn-taking may be provided (e.g. clockwise). As in the previous step, the information is recorded facilitating prediction and checking of the rule.
- This task can be extended by giving the groups the pattern and asking them to find the rule.

Resources: https://polypad.amplify.com/p#functions

Open Task

Oide

Oide

$\square \times \square = \square\square \div \square$

Using the digits 2 to 9, place a digit in each box to make two correct equations: one where the value is greater than 30 and one less than 30. You may reuse all the digits each equation.

Alternative: use each digit only once.

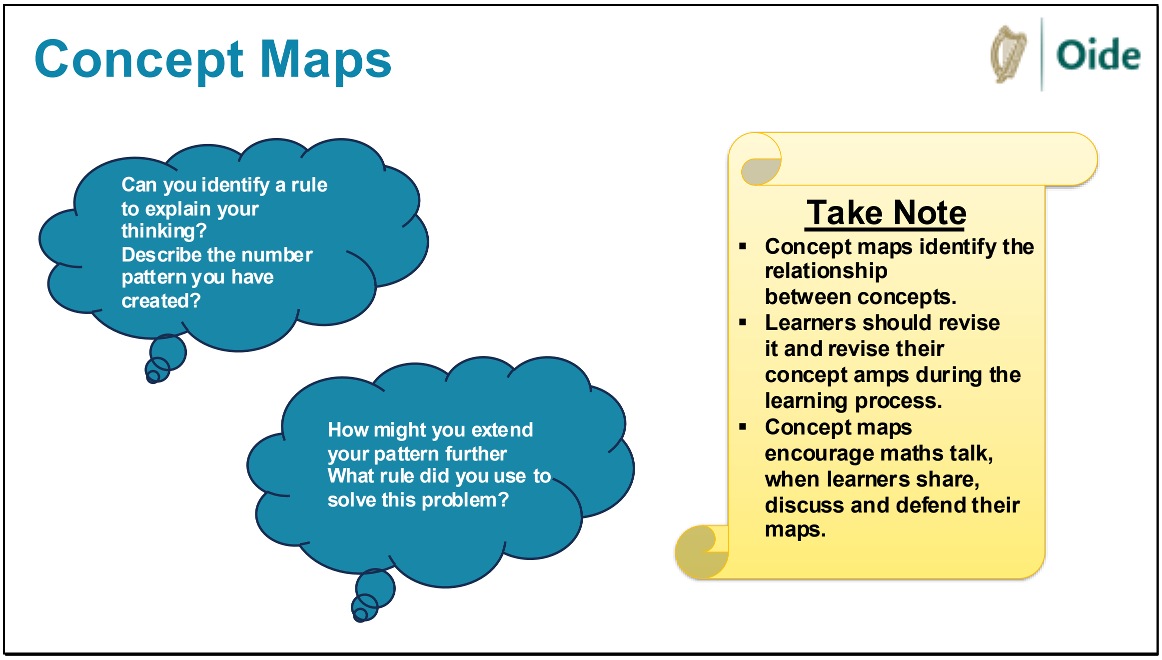
Adapted from openmiddle.com

Purpose of slide:
Engage in an open ended task.

Notes for teachers:

- Consider what makes this task an open ended task? (multiple pathways and multiple solutions)
- Is there more than one solution?
- Are there a set of rules or procedures to follow?
- What is the mathematical point of the task?
- Give children the opportunity to engage with the task.

Resources:
Access further open-ended tasks in the Micro-Maths section of the https://pmc.oide.ie/ website via the QR code.



Purpose of slide:
To explore concept maps as a form of assessment.

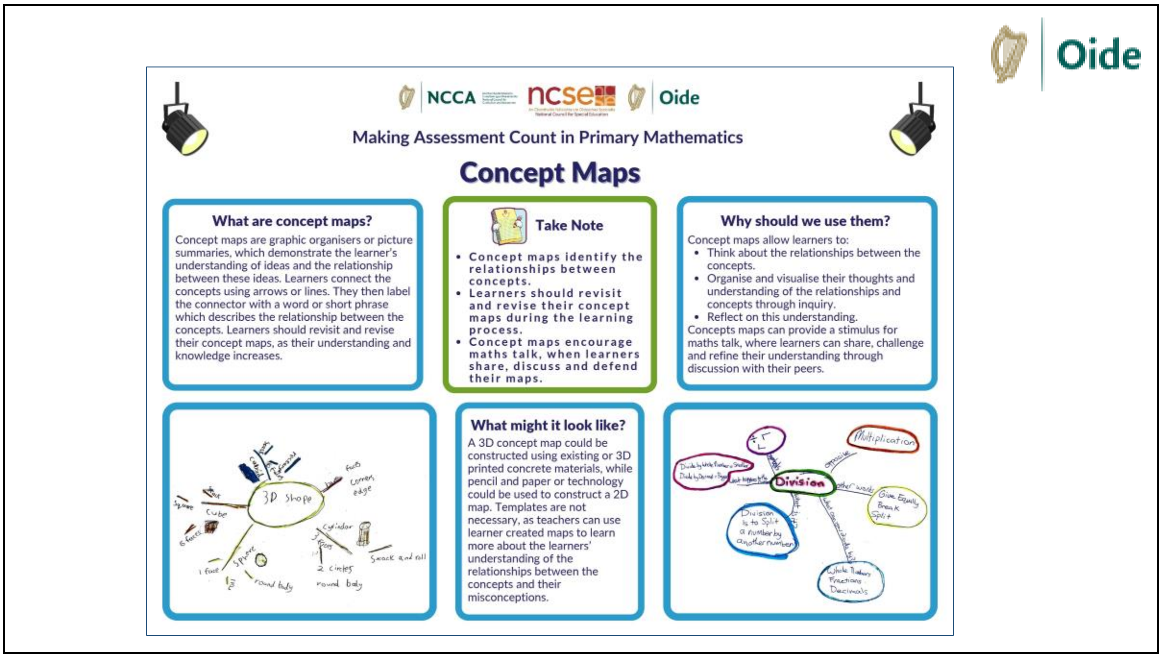
Notes for teachers:

- Concept maps are visual representations of information. They can take the form of charts, graphic organisers, tables, flowcharts, Venn Diagrams, or timelines. Concept maps are especially useful for students who learn better visually, although they can benefit any type of learner. They are a powerful study strategy because they help you see the big picture: by starting with higher-level concepts, concept maps help you chunk information based on meaningful connections.
- In other words, knowing the big picture makes details more significant and easier to remember.
- Concept maps work very well to see and understand relationships between different concepts. Visually then it is easier to analyse information and compare and contrast.

- <u>Making and using concept maps:</u> Making one is simple. There is no right or wrong way to make a concept map. The one key step is to focus on the ways ideas are linked to each other. For a few ideas on how to get started, take out a sheet of paper and try following the steps below:

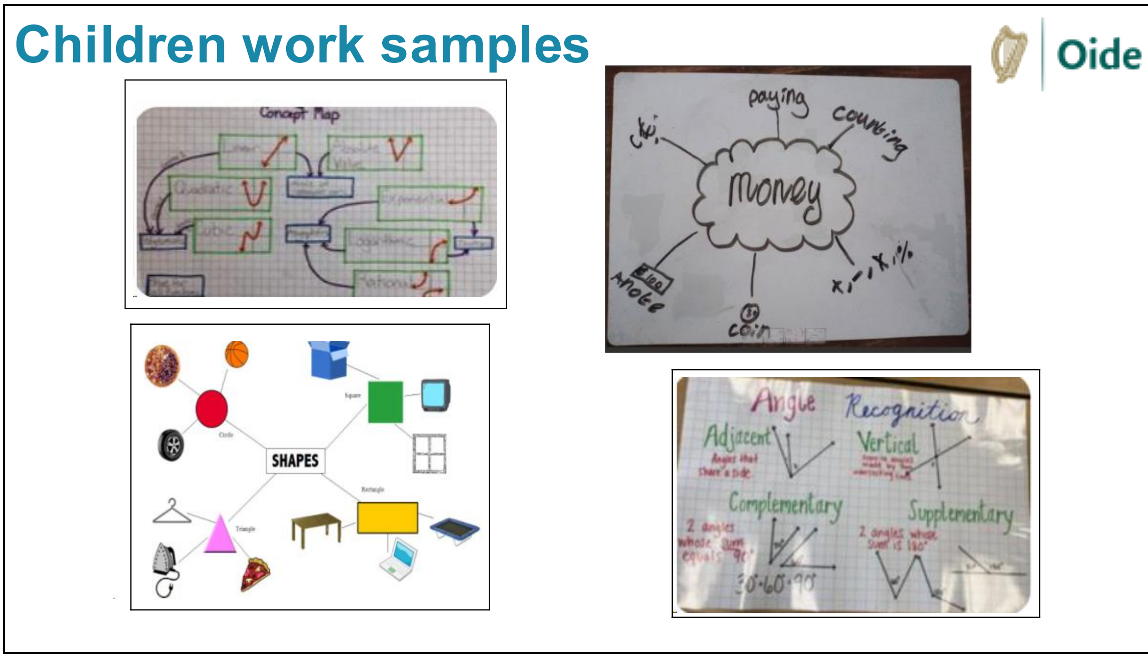
- Identify a concept.
- From memory, try creating a graphic organiser related to this concept. Starting from memory is an excellent way to assess what you already understand and what you need to review.
- Concept maps could also be used as a tool within maths journals.

Resource:
NCCA Support Materials: 'Making Assessment Count in Mathematics' available on https://pmc.oide.ie/



Purpose of slide:
To further examine Concept Maps

Notes for teachers:
Use the NCCA support document 'Making Assessment Count in Mathematics' available on https://pmc.oide.ie/ for information about Concept Maps.



Purpose of slide:
To provide examples of children's concept maps.

Notes for teachers:

- As mentioned in the previous slide, concept mapping is designed to dramatically depict the suggested relationship between different ideas and concepts. Often written within circles or boxes and described using keywords, conceptsare linked by words or phrases that explain the connection between these different ideas.
- Here are some examples. The concept on the top right is done on a mini whiteboard. A picture can be taken to record this form of self-assessment.
- The bottom left is done through use of IT.
- Concept maps can take many forms. They are a form of self-assessment.
- Involving the learner/s in the whole process of assessment gives clarity and direction to teaching and increases motivation in learners. Involving learners in assessing their own and each other's work affords the teacher greater insight into learner motivation and progress and is a source of valuable data for reporting to parents.