

 **Oide** Taro leis an bhFoghlaim Ghairmiúil i measc Ceannaircneirí Scoile agus Múinteoirí Supporting the Professional Learning of School Leaders and Teachers

Muinín Stage 2 Algebra Expressions & Equations



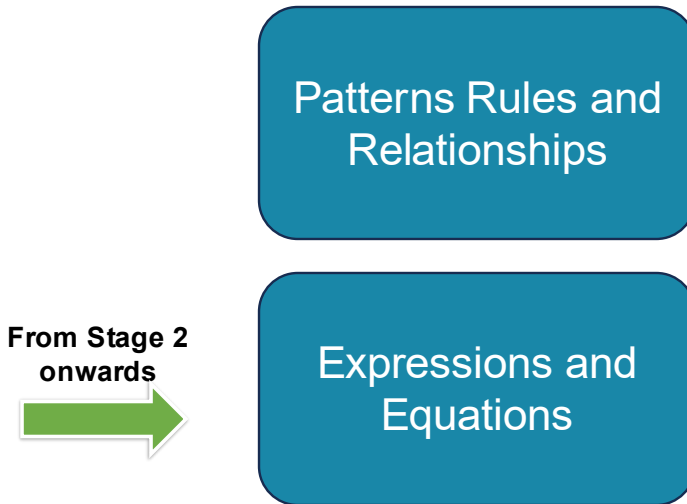
The photograph shows two children at a table. One child is writing on a piece of paper with a marker. The paper has handwritten text: '1st row: 3 sticks', '2nd row: 6 sticks', '3rd row: 9 sticks', and '4x3'. On the table, there are several wooden sticks arranged to form a large triangle composed of smaller triangles.

Purpose of slide:

Introductory slide for presentation of Stage 1 Algebra – Patterns, Rules & Relationships.

Strand: Algebra

Strand Units



Purpose of slide:

To provide an overview of the Algebra strand.

Notes for teachers:

- Go to page 19 of curriculum document.
- There are two strand units within the strand of Algebra.
- **Stage 1 To Stage 4 :** Patterns, Rules and Relationships.
- **From Stage 2 onwards:** Expressions and Equations.
- Why are the strand units presented in this way?
 - Younger students are still developing their abstract thinking skills. The focus is on building a strong foundation in patterns as a great deal of Algebra concerns pattern in one way or another, from making and describing patterns, to making rules and finding general rules to making a rule to predict the results of patterns. Implicit in this is the notion of relationships. As students progress through the curriculum, they will gradually be introduced to algebraic concepts, including expressions and equations, in a way that builds on their existing knowledge and skills.

Expressions & Equations

Progression across the stages



Learning Outcomes for Expressions and Equations Strand Unit			
Stage 1: Junior and senior infants	Stage 2: First and second classes	Stage 3: Third and fourth classes	Stage 4: Fifth and sixth classes
<i>Through appropriately playful and engaging learning experiences, children should be able to</i>			
	interpret the meaning of symbols or pictures in number sentences.	represent and express problems with known and unknown values in different ways to include the use of appropriate letter-symbols or words .	articulate, represent and solve mathematical situations through the use of expressions and equations that include letter-symbols .



Purpose of slide:

To explore the progression across the stages in the strand unit Expressions & Equations.

Notes for teachers:

- Notice the progression along the stages.
- Note how language, knowledge and skills are developed from stages 1 to 4.
- Knowledge of progression is necessary so that we can adapt and extend our teaching based on the knowledge we have of the children in front of us.
- Algebra permeates all strands of this curriculum and is at the heart of mathematics.
- There are two types of algebra, formal algebra (equations) and algebraic thinking/reasoning.
- We have up until now only encountered formal algebra when entering post primary school and in general we had no basic understanding/experience of algebraic thinking.
- The emphasis for Algebra in primary school should be on the development

of algebraic thinking.

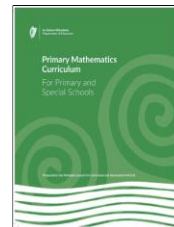
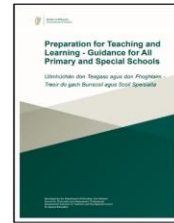
- Algebraic thinking is essential for students to engage meaningfully with formal algebra.
- Looking at the learning outcomes we can see how each stage builds upon the last, fostering a rich understanding of symbols and their mathematical significance.

Learning Outcome

Recorded Preparation



Learning Outcome:
Through appropriately playful and engaging learning experiences, children should be able to:
interpret the meaning of symbols or pictures in number sentences.



Purpose of slide:

To highlight the learning outcome as the starting point for preparation for teaching and learning.

Notes for teachers:

- Many children enjoy, and experience success in mathematics in primary school, until the introduction and manipulation of abstract symbols in secondary school.
- Algebra is a form of expression; it's a way to communicate.
- If I'm going to communicate something, I need to understand what it is. No more should children be presented with $a + b = b + a$ until they have a robust understanding of equivalence, the underpinning properties (commutative...) and why and how we use symbols.
- It develops over time.
- About 70% of questions in JC maths exam requires algebra. This is only 3 years from when they leave us!
- Key Findings of the Early Algebra Movement
 - Understanding Symbols in Maths - Using symbols in mathematics

these symbols mean. This means moving beyond just memorizing rules, such as $(a + b = b + a)$ or solving for (x) in equations like $(3x + 7 = 34)$.

- Learning Takes Time - Understanding algebra takes time and practice. Students need many chances to hear, see, and express their ideas about maths.
- What Students Need to Learn - Children should understand the idea of equivalence, which means knowing when two expressions are equal. It's important for students to know why we use symbols in math and how they help us solve problems.

Algebra

Maths Concepts



Stage 2 (1st & 2nd Class)	
Learning Outcomes	Interpret the meaning of symbols or pictures in number sentences.
Mathematical concepts	Real-life situations can be expressed using manipulatives, diagrams, and word and number sentences.
	An equals sign (=) conveys equality, whereas $\neq$, $<$ and $>$ convey inequality.
	In number sentences (equations), symbols can stand for a request to do something (+, -, $\times$, $\div$), they can express a relationship (=, $<$, $>$, $\neq$), or they can be something that is unknown or varies.
	In a number sentence, number facts can be applied to help find an unknown value.



Purpose of Slide:

To highlight the Maths Concepts which underpin the learning outcome for Stage 2 Expressions & Equations.

Notes for teachers:

- The Maths Concepts are the key mathematical ideas that underpin each learning outcome.
- The Maths Concepts may be useful in identifying a Focus of New learning when preparing for teaching and learning.
- Take a few moments to explore the Learning Outcomes and the Maths Concepts on the NCCA Maths Toolkit.

Resources required:

<https://curriculumonline.ie/primary/curriculum-areas/mathematics/algebra/> = QR code

Equivalence



$$8 + 4 = \square + 5$$

Operational view of
Equality

$$8 + 4 = 12 + 5$$
$$8 + 4 = 17 + 5$$

Relational view of
Equality

$$8 + 4 = 7 + 5$$

Purpose of slide:

To introduce a learning experience.

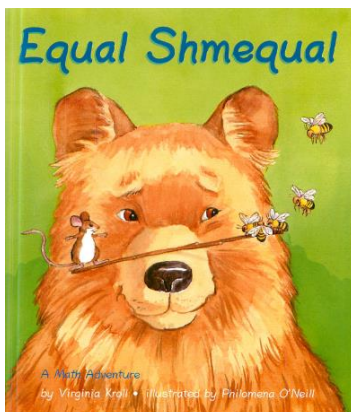
Notes for teachers:

- Repeated studies have found that fewer than 10% of children between ages 6-12 correctly complete this equation.
- In one 1999 study, not one of 145 12-year-old participants solved it correctly (Faulkner, Levi & Carpenter, 1999).
- Students are immersed in $3 \times 8 = \underline{\quad}$; $6 + 2 = \underline{\quad}$ and it becomes automatic to think “the answer comes next”.
- This misconception is highly problematic when students encounter complex expressions and equations in senior primary and in secondary school.
- To investigate your own pupils’ understandings of equality, present them with the above problem and record their responses.
- The above (incorrect) responses indicate an operational view of equality:

- $8 + 4 = 12 + 5$
- $8 + 4 = 17 + 5$
- When children give responses of 12 or 17, it is because they believe that the equals sign means the answer is. Consequently, it seems logical to such children that when you see an equals sign you should perform the calculation that precedes the equals sign and that the number to the right of the equals sign is the answer to that calculation. This 'operator' view is the result of an overemphasis on problems of the form $a + b = q$
- The above (correct) response indicates a relational view of equality: $8 + 4 = 7 + 5$
- Children with a relational view of equality believe that the equals sign means 'is the same value as'. Such children understand that the amounts either side of the equals sign are relationally the same. They then search for a value that when placed in the frame will result in both sides of the equals sign balancing each other.

Story

Equal Shmequal



Purpose of slide:

To introduce a learning experience: Book - [Equal Shmequal](#).

Notes for teachers:

Focus on the meaning of the language-what does equal mean? Before we can use it, we have to understand it.

This is a story that offers a fun look at equality in the real world as it relates to play. Using it as a stimulus, learners are introduced to the concept of equality and balance. The seesaw is an analogy to develop children's relational understanding of equality among children.

The story is about a group of animals who decide to play a friendly game of tug of war but struggle to make the sides equal to make sure the game is fair. They solve the problem by distributing animals either side of a see-saw until both sides balance. Balancing different sized animals reinforces the notion that

things can be the same value but at the same time look different.

There are read aloud versions available on YouTube which allows the children to see the images as the story is read. As the Equal Shmequal story is being read, children can communicate their own predictions regarding who would win using their hands to demonstrate a notion of balance. This reinforces the idea that the value of quantities either side of the equals sign must balance each other.

Resources required:

<https://www.youtube.com/watch?app=desktop&v=vOGG5MF3i54>

Teddies at the Restaurant

10 Teddies went to a restaurant.
Show how the teddies may have sat at
the tables.



Purpose of slide:

To examine a learning experience.

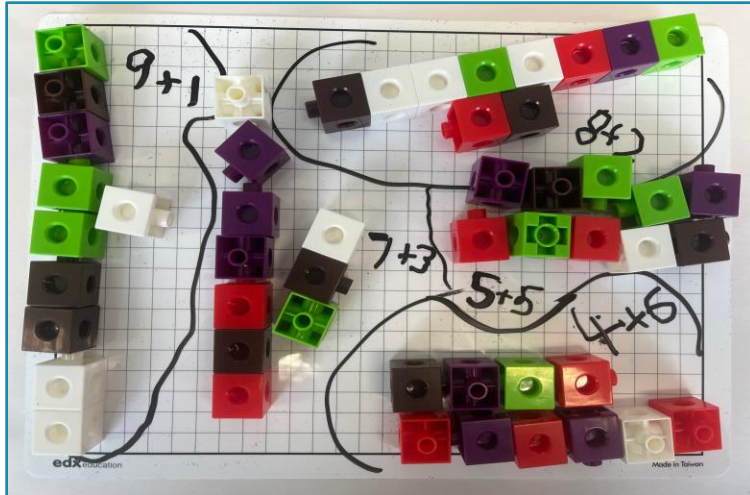
Notes for teachers:

- Explore the story of 10 / Fact Family of 10/ Facts of 10 using concrete materials.
- Ask the children to think of all the ways they can write 10 (this can happen in a single lesson and the other tasks below on a subsequent day/s).
 - Children will likely use just two numbers e.g. $2 + 8 = 10$ but some children may give examples using multiple numbers combining to make 10 (as shown on the slide e.g. $3 + 3 + 3 + 1 = 10$).
 - The multiple ways to make 10 should be available as a display.
 - Make a display of '10':
 - 10 can be represented in various ways, including:

- 10 is the same (amount) as: e.g, $10 + 0$, $9 + 1$, $8 + 2$, $7 + 3$, $6 + 4$, $5 + 5$, $4 + 6$, $3 + 7$, $2 + 8$, $1 + 9$, $0 + 10$, $3 + 3 + 3 + 1 = 10$
- 10 is equal to: $10 + 0$, $9 + 1$, $8 + 2$, $7 + 3$, $6 + 4$, $5 + 5$, $4 + 6$, $3 + 7$, $2 + 8$, $1 + 9$, $0 + 10$, $3 + 3 + 3 + 1$
- These expressions show that all these combinations equal 10.
- Consider what equals means:
 - Write the word 'equal' on the chart/smartboard. Have students tell you what 'equal' means. Brainstorm ideas and record these.
 - Ask if students know how to write "is equal to" using a symbol. Introduce =
 - Have students pair share the task of writing the complete equations.
 - If = means is equal to or the same amount as, can we write the equation like this:
 - $9 + 1 = 8 + 2$
 - $10 = 9 + 1$
 - $9 + 1 = 10$
 - Do all of these equations use equals properly?
 - Have children pair shared how many equations they can write where the total comes first or at the end, or where they are showing equality between two additions.

Equality

Story of 10



Purpose of slide:

To examine a learning experience.

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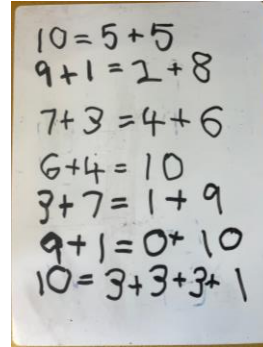
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Equality

Story of 10



$$\begin{aligned}9 + 1 &= 8 + 2 \\10 &= 9 + 1 \\9 + 1 &= 10\end{aligned}$$



Handwritten list of equations showing various ways to make 10:

$$\begin{aligned}10 &= 5 + 5 \\9 + 1 &= 2 + 8 \\7 + 3 &= 4 + 6 \\6 + 4 &= 10 \\3 + 7 &= 1 + 9 \\9 + 1 &= 0 + 10 \\10 &= 3 + 3 + 3 + 1\end{aligned}$$

Purpose of slide:

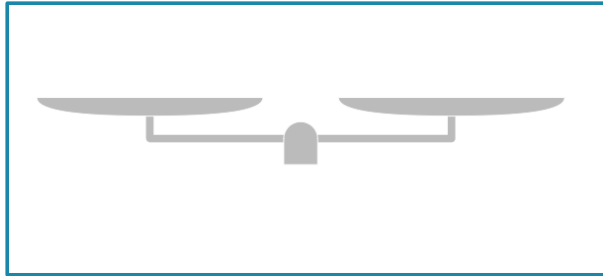
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Equivalence



is the same as



Purpose of slide:

- To highlight the importance of language.
- To demonstrate the use of a pan balance.

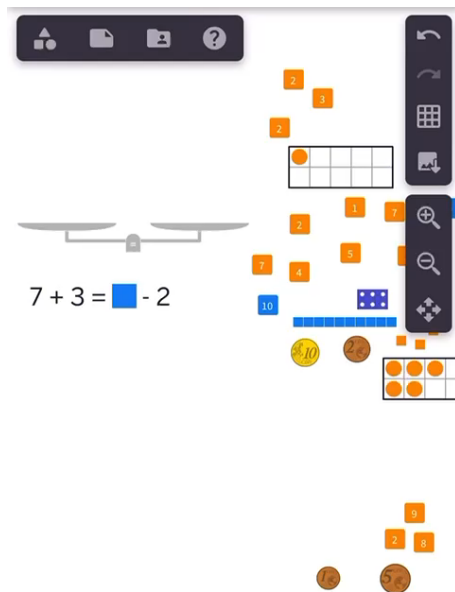
Notes for teachers:

- Children must develop a relational understanding of equality.
- Understanding the Equals Sign - Many children interpret the equals sign (=) as "makes" or as a cue to "find the answer." It's important to clarify that it means "is equal to," indicating that both sides represent the same number.

Resources required:

<https://www.didax.com/apps/math-balance/>

Equivalence



Purpose of slide:

To use online tools to explain equivalence.

Notes for teachers:

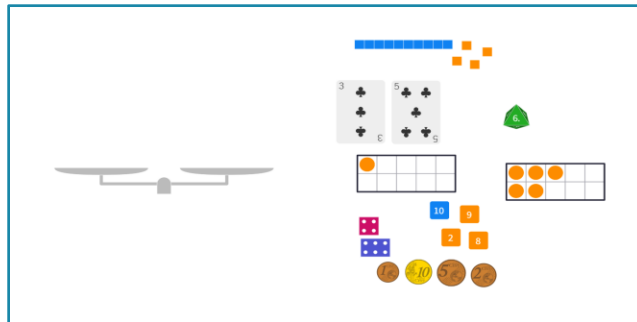
- Using Pan Balances - Children can use pan balances to solve number sentences by finding the value that balances both sides.
- On the slide you will find the balance on Polypad which could be used as a tool to explore number (also available in a simpler version on Didax).
- The Polypad balance works by value, not weight, which allows students to focus on the numerical relationships rather than physical weights.
- When working with this resource students can choose which one makes sense to them (Child agency). This helps students understand how different combinations can equal the same value.
- Number sense cannot be imposed. It is developed through exploration and investigation. Manipulating a balance is key in developing the understanding of equivalence before moving to the abstract formal algebra symbol $=$.
- QR code on screen is a link to this balance on Polypad.

Resources required:

- [Polypad](#)
- [Didax Math Balance](#)

Open number sentences

$$4 + 8 = \star + 7$$



Purpose of slide:

To examine a learning experience.

Notes for teachers:

- Ask children what number can complete this number sentence? $4 + 8 = _ + 7$
- Use different symbols to represent the unknown – we have used a star in this example.
- Allow children to think pair share and take all solutions.
- Review, what does = mean? The same as, equals to. Everything on the left must balance with everything on the right.
- Allow for robust discussion and make sure to dispel any misconceptions, e.g. whether there is more than one correct solution.
- Repeat with $9 - 3 = _ + 2$

- Encourage pupils to explore number sentences using a balance (Polypad or Didax).

Resources required:

<https://polypad.amplify.com/p/fet5Wh0fLo3Vw>

<https://www.didax.com/apps/math-balance/>

Open Number Sentences



$$4 + 8 = \text{cloud} + 7$$

$$347 + 18 = \text{orange square} + 19$$

$$199 - \text{heart} = 201 - 14$$

What do you notice about the right side and the left side?

How could that help us solve this?

Purpose of slide:

To examine a learning experience.

Notes for teachers:

- Progress to the number sentence $347 + 18 = _ + 19$.
- Instruct the children not to add 347 and 18. Instead, encourage them to think about what number would make the equation balance.
- Have them discuss their thoughts with a partner or group without calculating the total. Afterwards, ask the children to verify their answers using calculators for the equation $199 - _ = 201 - 14$.
- As they work with larger numbers, emphasise the importance of relational thinking over simple computation. This approach fosters their reasoning skills.
- Encourage children to utilise properties of operations or known facts to help solve the open sentences.
- Note that exploring the properties of operations is part of the patterns, rules, and relationships strand unit in the curriculum.
- Linking back to 'learning maths is the study of relationships,

connections and patterns' (from rationale of curriculum) This will lead to children developing strategies such as compensating.

Equality

Number sentences



There are 9 bears in all. How many are in the cave?



$$\begin{aligned}9 &= 5 + \square \\5 + \square &= 9 \\9 - 5 &= \square \\9 - \square &= 5\end{aligned}$$

Purpose of slide:

To examine a learning experience.

Notes for teachers:

- This concept is similar to Steve Wyborney's Splat activities seen in the Money Muinín workshop, there are many number Splats available on his site.
- Bears in a cave activity: This learning experience is from Kentucky Centre for Maths. You can download activities as Google Slides.
 - There are 9 bears in all.
 - Children are shown a range of number sentences that represent the task. Ask: which one represents this task best? Do they all work? How many are in the cave? Which sentence represents the story? Why?
- (Useful language: Missing addend / Missing subtrahend (What is the

meaning of subtrahend? sub· tra· hend-a number that is to be subtracted from another number).

Resources required:

https://www.kentuckymathematics.org/vr_structuring.php

Draw or Write an Equation



The Snozzcumbers

The BFG had 10 Snozzcumbers.
He ate some and now he has 4 left.
How many did he eat?



Purpose of slide:

To examine a learning experience.

Notes for teachers:

- Ask children to draw or write an equation to represent the story.
- Ask: Why can the number sentence/s be used to solve the problem?
- Can link this back to counting activity (bucket count) when we asked children to write a number sentence to match the count.

Draw or Write a Story



$$9 = \square + 5$$

$$11 = \square - 2$$

$$\square = 23 + 5$$

Purpose of slide:

To examine a learning experience.

Notes for teachers:

- If a child can write a number sentence they can turn this around to come up with a story around an equation e.g., can they work it backwards e.g. There were 9 girls in 2nd Class, 5 of them were in school, how many were absent?

True or False?

10 + 4 is equal to 15 - 3	9 - 2 is equal to 8 - 3	1 + 3 is equal to 2 + 2
3 + 1 is the same as 5 - 1	6 + 3 = 5 + 4	8 + 1 + 0 = 0 + 1 + 8
9 - 2 is equal to 8 - 3	12 + 5 is the same as 17 - 1	23 + 1 is the same as 15 + 8



Purpose of slide:

To examine a learning experience.

Notes for teachers:

- Children may not have prior experience with true/false number sentences, but introducing them is easy.
- Discussion: Engage children in a conversation about what makes a number sentence true or false.
- Example: Provide a simple number sentence and ask if it's true or false. Start with basic calculations involving a single digit on the right side of the equal sign, e.g. $1 + 3 = 4$
- Concrete Materials: Allow children to check their answers using concrete materials. If a sentence is identified as false, discuss the reasoning behind it and clarify the meaning of the equals sign. Use a balance symbol as a visual aid and invite children to represent comparisons with their materials.

- Proof: Encourage children to prove which statements are true and which are false.
- Repeat: Use a second set of expressions for reinforcement. Consider using a true/false board for additional engagement.

Solve me Puzzles

Number Line

The helper thinks of a secret number under 10.

Someone else calls out a number under 10.



The counter starts at 0. The helper moves it along the number line for their secret number of squares and the other number of squares put together.

Can you work out the secret number?

I wonder if you can make up other games using the number line?

nrich.maths.org/roadshow

Super Shapes

Each of the following shapes has a value:

△ = 7 □ = 17

Can you discover the values of the other shapes using the information below?

(a) △ + ○ = 25

(b) □ + △ + △ + △ = 51

(c) △ + △ + △ + △ + △ + □ + □ = 136

(d) △ + △ + △ = 48

(e) △ + △ + △ + □ + △ + △ + △ = 100

nrich.maths.org/1038
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Purpose of slide:

To examine learning experiences - Three activities (Secret Jumps, Super Shapes and Secret Number).

Notes for teachers:

- Explanation of one of these tasks - Secret number:
- This is a game for two players with a calculator.
Annie and Ben are playing. Annie puts her secret number into the calculator without showing Ben.
Annie then asks Ben, "What do you want to add?"
Ben tells Annie the number he wants to add: "I want to add four."
Annie presses the 'add' button and then the four button. The calculator now shows '4'. Annie gives the calculator to Ben.
- Ben presses the 'equals' button, and the calculator gives the answer '10'.
- What was Annie's secret number?
How do you know?

- You could play this with a friend. If you work out your friend's secret number correctly, it is your turn to put in a secret number of your own. You could score a point for every one you get right.
- Now take time to look at the other 2 tasks on the slide via the links below

Resources required:

- Secret jumps on number line - <https://nrich.maths.org/5652&part=>
- Super shapes - <https://nrich.maths.org/1056&part=>
- Secret number – <https://nrich.maths.org/5651&part=>

Open Ended Task

$$\square = \square + \square = \square + \square + \square$$

Using the digits 1 to 9, place a digit in each box to create a true statement.



Alternative: use each digit only once.

Adapted from: <https://www.openmiddle.com/equivalent-statements/>



Purpose of slide:

To examine a learning experience.

Notes for teachers:

- This is an example of a task taken from openmiddle.com.
- Other open ended tasks for Algebra are available in the Resources section of the pmc.oide.ie website.

Resources required:

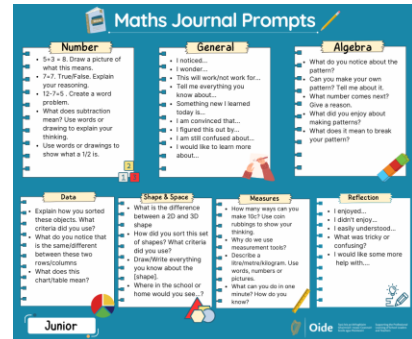
<https://www.openmiddle.com/>

<https://pmc.oide.ie/resources/micro-maths/>

Maths Journal

Something new
I learned today
is

Tell me
everything you
know about the
word Equal



Purpose of slide:

To provide reflective prompts to use in class.

Notes for teachers:

- Journals are useful for both teachers and learners to assess attitudes, knowledge and skills.
- Children can keep track of their thinking and understanding in the journal.
- Journals can contain general observations about Maths or can be more specific and focus on a particular concept.
- On the slide are two journal prompts which can be used in class. The first one focuses on the child's disposition and can be used across all stand units.
- Journal prompts - I OBSERVED, I DISCOVERED, I CHECKED, I PROVED
 - Today I enjoyed... - general prompt to get the children thinking about maths and the areas that they are curious about.
 - How many ways can you make €1? Use words or drawings to explain your thinking. – teacher decides on an appropriate amount.